61412

ROLL No. | | | | | |

TEST BOOKLET No. **158**

TEST FOR POST GRADUATE PROGRAMMES

STATISTICS

Time: 2 Hours

Maximum Marks: 450

INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.
2. Write your Roll Number in the space provided on the top of **this page**.
3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the **Answer Sheet**. Darken the appropriate bubbles with a **Ball Point Pen.**
4. The paper consists of 150 objective type questions. All questions carry equal marks.
5. Each question has four alternative responses marked **A, B, C** and **D** and you have to **darken** the bubble fully by a **Ball Point Pen** corresponding to the correct response as indicated in the example shown on the Answer Sheet.
6. Each correct answer carries **3** marks and each wrong answer carries **1** minus mark.
7. Please do your rough work only on the space provided for it at the end of this Test Booklet.
8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.
9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of such unforeseen happenings the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

SEAL

STATISTICS

1. For a matrix A, if $A^2 = A$, then matrix A is called

(A) orthogonal
(B) idempotent
(C) nilpotent
(D) involutory

2. The largest eigen value of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ is

(A) 3
(B) 5
(C) 6
(D) 1

3. The matrix $\begin{bmatrix} -3 & 2 \\ 2 & -5 \end{bmatrix}$ is

(A) negative definite
(B) positive definite
(C) negative semi definite
(D) indefinite

4. $\lim_{n \to \infty} \left(\sqrt{n+1} - \sqrt{n} \right)$ is equal to

(A) 1
(B) 2
(C) ∞
(D) 0

5. The inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$ is

(A) $\begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix}$
(B) $\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

6. The rank of the matrix $\begin{bmatrix} 1 & -2 & 2 \\ 2 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$ is

(A) 3
(B) 2
(C) 1
(D) 4

7. If $n_{C_{12}} = n_{C_8}$ then the value of n is

(A) 30
(B) 20
(C) 12
(D) 6

8. $\int \frac{1}{\sqrt{x}} dx$ is

(A) $\frac{1}{2\sqrt{x}}$
(B) $2\sqrt{x}$
(C) $x^{3/2}$
(D) $\frac{1}{\sqrt{2x}}$

9. The square root of $5+12i$ is

(A) $3+2i$
(B) $3-2i$
(C) $\pm(3+2i)$
(D) $(2i-3)(2i+3)$

10. If $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, then dy/dx is

(A) $\frac{1-x^2}{(1+x\ )}$
(B) $\frac{1-x}{(1+x^2)}$
(C) $\frac{1}{(1+x^2)}$
(D) $\frac{-1}{(1+x^2)}$

11. Matrix A is said to be orthogonal if

(A) $|A|=1$
(B) $AA^{-1}=I$
(C) $AA^T=I$
(D) $|A|=0$

12. The variance of 4, −5, −1, 0 and 2 is

(A) 5/46
(B) 46
(C) 9
(D) 46/5

13. The harmonic mean of 10, 20 and 30 is

(A) 1/20
(B) 11/180
(C) 180/11
(D) 20

14. The following is the probability distribution of a random variable X

x	−2	−1	0	1	2
$p(x)$	0.05	0.15	0.40	0.30	0.10

Then $E(X)$ is

(A) 0.30
(B) 0.25
(C) −0.25
(D) 0.20

15. In a class 72 students offered Physics, 48 students took Chemistry and 34 students took Mathematics. Of these 24 took both Chemistry and Mathematics, 15 took both Chemistry and Physics and 12 took both Physics and Mathematics. If 7 students offered all the three subjects, then the total number of students in the class is

(A) 154
(B) 110
(C) 96
(D) 198

16. A and B are two events. The probability that exactly one of these two events will occur is

(A) $P(A)+P(B)$
(B) $P(A)+P(B)-P(A\cap B)$
(C) $P(A\cap B)+P(A\cup B)$
(D) $P(A\cap B^c)+P(A^c\cap B)$

17. It is known that 1% of the items produced in a factory are defective. A random sample of 100 units is selected from a day's production. The probability of getting exactly two defectives in the sample is approximately equal to

(A) $1/2e$
(B) $e^{-2}/2$
(C) 0.001
(D) $2e^{-2}$

18. A $r.v.$ X has the p.d.f $f(x) = \dfrac{e^{-x/\beta}}{\beta}, x > 0$. Then $E(X)$ is

(A) 2β (B) β
(C) $1/\beta^2$ (D) $1/\beta$

19. Let $X_1, X_2, \ldots X_n$ be a random sample from the distribution with $p.d.f.$ $f(x,\theta) = \theta e^{-\theta x}$, $x > 0$. Then $m.l.e.$ of θ is

(A) $\bar{X}$ (B) $X_{(1)}$
(C) $1/\bar{X}$ (D) $\bar{X}e^{\bar{X}}$

20. The $p.m.f.$ of a $r.v.$ X is $p(x) = pq^x$, x=0,1,2,.... Then $E(X)$ is

(A) q/p (B) q/p^2
(C) $1/p^2$ (D) $1/p$

21. The population of a city in the beginning of 2005 was 100 lakhs and in the beginning of 2007 was 169 lakhs. Then the annual average increase of population is

(A) 30% (B) 69%
(C) 34.5% (D) 35%

22. The mean weight of 150 students in a certain class is 60 kg. The mean weight of the boys from the class is 70 kg, while that of the girls is 55 kg. Then the number of girls in the class is

(A) 70 (B) 90
(C) 60 (D) 100

23. A cyclist covers his first 20 km. at an average speed of 40 km.p.h, another 10 km. at 10 km.p.h. and the last 30 km at 40 km.p.h. Then the average speed of the entire journey is

(A) 20 km.p.h. (B) 26.67 km.p.h.
(C) 28.24 km.p.h. (D) 24 km.p.h.

24. Let X and Y be two related variables. The two regression lines are given by $x-y+1=0$ and $2x-y+4=0$. The correlation coefficient between X and Y is

(A) 0.71 (B) 0.25
(C) 0.63 (D) 0.58

25. The *m.g.f.* of a *r.v.* X is $\exp(t^2)$. Then the distribution of X is

(A) Laplace (B) $N(0,2)$
(C) Cauchy (D) $N(0,\sqrt{2})$

26. Let $F(x)$ be the *d.f.* of a continuous *r.v.* X. Then $F(X)$ follows

(A) exponential (B) normal
(C) uniform (D) None of the above

27. Two fair dice are thrown. What is the probability that the sum of the numbers is greater than 8?

(A) 20/36 (B) 15/36
(C) 5/18 (D) 7/36

28. Let $X_1 \sim b(n_1,p)$ and $X_2 \sim b(n_2,p)$ and X_1 and X_2 are independent. Then the distribution of X_1+X_2 is

(A) Bernoulli (B) Binomial
(C) Poisson (D) None of the above

29. Let $X \sim \chi^2_m$, $Y \sim \chi^2_n$, X and Y are independent, then the distribution of X/Y is

(A) Gamma (B) $\text{Beta}_1(m,n)$
(C) $\text{Beta}_2(m,n)$ (D) Chi-square

30. If $P(A)=0.3$, $P(B)=0.3$ and $P(A|B)=0.2$, then $P(B|A)$ is

(A) 0.02 (B) 0.30
(C) 0.10 (D) 0.20

31. Identify the odd item in the following:

(A) Local control
(B) Replication
(C) Randomisation
(D) Confounding

32. The purpose of replication is

(A) to estimate the missing observations
(B) to eliminate the interaction effect
(C) to average out the influence of chance factors
(D) to average out the effect of treatments

33. Power of a test is the probability of

(A) accepting H_0 when H_1 is true
(B) accepting H_1 when H_0 is true
(C) rejecting H_0 when H_0 is true
(D) rejecting H_0 when H_1 is true

34. If X and Y are *i.i.d.* geometric *r.v's*, then the distribution of $X+Y$ is

(A) geometric
(B) Poisson
(C) negative binomial
(D) compound Poisson

35. If X and Y have a bivariate normal distribution, then the conditional distribution of X given Y is

(A) Cauchy
(B) standard normal
(C) bivariate normal
(D) normal

36. The quantity $\beta_2 = \mu_4 / \mu_2^2$ is a measure of

(A) relative dispersion
(B) skewness
(C) kurtosis
(D) correlation

37. The probability distribution underlying the control limits of C-chart is

(A) normal
(B) binomial
(C) Poisson
(D) Chi-square

38. S-chart is used for controlling

(A) process mean
(B) process fraction defective
(C) number of defects
(D) process dispersion

39. Among the following, which test is a non-parametric test?

(A) Chi-square test
(B) t-test
(C) Z-test
(D) F-test

40. The degrees of freedom for the t-test for the equality of two population means with 10 observations on X and 8 observations on Y is

(A) 15
(B) 16
(C) 17
(D) 18

41. The degrees of freedom for the 'error sum of squares' in an RBD with 4 treatments and 5 blocks is

(A) 15
(B) 12
(C) 14
(D) 16

42. Under SRSWR, variance of the sample mean is

(A) $\left(1-\frac{1}{N}\right)\frac{S^2}{n}$
(B) $\left(1-\frac{n}{N}\right)\frac{S^2}{N}$
(C) $\left(1-\frac{n}{N}\right)\frac{S^2}{n}$
(D) $\left(1-\frac{1}{N}\right)\frac{S^2}{N}$

43. Neyman - Pearson Lemma is used for

(A) finding the UMV estimator.
(B) testing a simple hypothesis against a simple alternative hypothesis.
(C) testing a simple hypothesis against a composite alternative hypothesis.
(D) testing a composite hypothesis against a composite alternative hypothesis.

44. If X and Y are independent Gamma variates, then the distribution of $Z = \frac{X}{X+Y}$ is

(A) Gamma
(B) Chi-square
(C) Beta
(D) normal

45. If $\varphi_X(t)$ is a characteristic function , then $\varphi_X(0)$ is

(A) 0 (B) ∞
(C) 1 (D) $E(X)$

46. If X is binomial with parameters n and $p=1/2$, and if $P(X=4)=P(X=5)$, then

(A) $n=6$ (B) $n=8$
(C) $n=10$ (D) $n=9$

47. For a binomial distribution:

(A) Mean=Variance (B) Mean < Variance
(C) Mean > Variance (D) Variance= $(\text{Mean})^2$

48. Let $X \sim N(0,1)$. Then $Y=X^2$ has the following distribution

(A) Cauchy (B) Chi-square
(C) Lognormal (D) Laplace

49. The *p.d.f.* of a *r.v.* is given by $f(x)=\frac{1}{2}\exp(-|x|),\ -\infty<x<\infty$. The distribution is called

(A) negative exponential (B) Weibull
(C) logistic (D) Laplace

50. The necessary and sufficient condition for the system of equations $AX=B$ to be consistent is

(A) $\rho(AB)\geq\rho(A)$ (B) $\rho(AB)\leq\rho(A)$
(C) $\rho(AB)=\rho(A)$ (D) $\rho(AB)\neq\rho(A)$

51. Given $n=5$ and $\sum_i d_i^{\,2}=2$, then the Spearman's rank correlation coefficient is

(A) 0.81 (B) 0.1
(C) 0.84 (D) 0.9

52. The characteristic function of a random variable X is $\frac{1}{1+t^2}$. Then X has the following distribution:

(A) Laplace
(B) Cauchy
(C) Exponential
(D) Uniform

53. Let $X \sim N(\mu, \sigma^2)$. Then $t = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n}$ is

(A) an unbiased estimator of σ^2
(B) a consistent estimator of σ^2
(C) least squares estimator of σ^2
(D) UMVUE of σ^2

54. Let $X \sim N(\mu, \sigma^2)$. For testing $H : \sigma^2 = \sigma_o^2$, the most powerful test is based on the distribution

(A) normal
(B) t
(C) Chi-square
(D) F

55. Laspeyere's formula for index number is

(A) $L = \frac{\sum p_1 q_o}{\sum p_o q_o} \times 100$
(B) $L = \frac{\sum p_o q_1}{\sum p_o q_o} \times 100$
(C) $L = \frac{\sum p_1 q_1}{\sum p_o q_o} \times 100$
(D) $L = \frac{\sum p_o q_o}{\sum p_1 q_o} \times 100$

56. Inversion formula is used

(A) to find the characteristic function given the $d.f.$
(B) to find the $c.f.$ given the $p.m.f.$ of a discrete distribution.
(C) to find the $d.f.$ from the $c.f.$
(D) to find the standard error of the estimator

57. The method of moving averages for time series is useful for

    (A) eliminating cyclical variation
    (B) estimating trend
    (C) removing seasonal fluctuation
    (D) studying the random component

58. A random variable X, with a pdf given by $f(x)=\begin{Bmatrix} \frac{1}{b-a}, 0<x<b; (b>a) \\ 0 \quad , otherwise \end{Bmatrix}$.

    The mean deviation about the mean is

    (A) $\frac{b+a}{4}$ (B) $\frac{b-a}{4}$
    (C) $\frac{b-a}{2}$ (D) $\frac{b+a}{2}$

59. Let $X_1, X_2 \ldots X_n$ be i.i.d Chi-square random variables with degrees of freedom 1. It is known that $S_n \sim \chi^2(n)$. Let $z_n = \frac{s_n - n}{\sqrt{2n}}$. Then $z_n \xrightarrow{L} z$ where z is

    (A) $N(0,1)$ (B) $N(0,1/n)$
    (C) $N(n, \frac{1}{\sqrt{2n}})$ (D) $N(0, \frac{1}{\sqrt{n}})$

60. Suppose one has a stereo system consisting of two main parts, a radio and a speaker. If the life time of the radio is exponential with mean 1000 hours and the life time of the speaker is exponential with mean 500 hours independent of the radio's life time, then the probability that the systems failure (when it occurs) will be caused by the radio failing is

    (A) 1/2 (B) 1/3
    (C) 2/3 (D) 3/4

61. If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, the probability $P(X=Y)$ is

(A) $e^{-(\lambda_1+\lambda_2)}\sum_{r=0}^{\infty}\frac{(\lambda_1\lambda_2)^r}{r!}$

(B) $e^{-(\lambda_1+\lambda_2)}\sum_{r=0}^{\infty}\frac{(\lambda_1\lambda_2)^r}{(r!)^2}$

(C) $e^{-(\lambda_1+\lambda_2)}\sum_{r=0}^{\infty}\left(\frac{\lambda_1}{\lambda_2}\right)^r\frac{1}{r!}$

(D) $e^{-(\lambda_1+\lambda_2)}\sum_{r=0}^{\infty}\left(\frac{\lambda_2}{\lambda_1}\right)^r\frac{1}{r!}$

62. If $X \sim N(0,1)$ and $Y \sim N(0,1)$ are independent variates, then (X/Y) and $X/|Y|$ have the same

(A) standard normal variates
(B) compound Poisson distribution
(C) standard Cauchy distribution
(D) compound binomial distribution

63. A random variable X having the $p.d.f$ $f(x) = \frac{\lambda}{\Pi[\lambda^2 + (x-\mu)^2]}$; $-\infty < x < \infty$; $\lambda > 0$ with location parameter μ and spread parameter $\lambda > 0$. The modal value of the distribution is

(A) $\mu + \lambda$
(B) μ
(C) $\mu - \lambda$
(D) Mode does not exist

64. When n, the sample size is larger than 30, the students t-distribution tends to

(A) normal distribution
(B) F-distribution
(C) Cauchy distribution
(D) Chi-square distribution

65. If (X,Y) is bivariate normal (3,1,16,25, 3/5), then $p(-3 < X < 3/Y = -4)$ is

(A) 0.3708
(B) 0.2734
(C) 0.1264
(D) 0.6442

66. Both the Central Limit Theorem (CLT) and the Weak Law of Large Numbers (WLLN) hold for a large class of sequence of *r.v.'s* $\{X_j\}$. For the CLT to hold for a sequence of uniformly bounded independent random variables, the condition $\frac{s_n^2}{n} \to \infty \ as \ n \to \infty$ is

(A) sufficient
(B) necessary and sufficient
(C) necessary
(D) $s_n^2 \longrightarrow 0$ is necessary and sufficient

67. A random variable X has the density function $f(x) = e^{-x}$ for $x \geq 0$. Then the Thebyshev's inequality is given as $P[|X-1| \geq 2] < \frac{1}{4}$, while the actual probability is

(A) e^{-2}
(B) e^{-1}
(C) e^{-3}
(D) e^{-4}

68. Let X_1, X_2......be *i.i.d* random variable's with common law $N(0,1)$. The limiting distribution of the random variable $W_2 = \sqrt{n}\,\frac{x_1 + x_2 + ... + x_n}{x_1^2 + x_2^2 + ... + x_n^2}$ is

(A) $N(0, \frac{1}{\sqrt{n}})$
(B) $N(0,1)$
(C) Gamma ($n/2$, $2/n$)
(D) Beta ($n/2$, $2/n$)

69. The numbers whose arithmetic mean is 12.5 and geometric mean is 10 are

(A) 20, 5
(B) 15, 10
(C) 12.5, 12.5
(D) 7, 18

70. The percentile range of a set of data is defined with

(A) $P_{90}+P_{10}$
(B) $P_{90}-P_{10}$
(C) $(P_{90}+P_{10})/2$
(D) $(P_{90}-P_{10})/2$

71. The Yule coefficient of association indicates the association between two attributes in terms of

(A) nature of association
(B) degree of association
(C) nature and extent of association
(D) limits of association

72. If $E(Y/X=x) = \frac{-1}{2}x+2$, for every x and $E(X/Y=y) = \frac{-3}{4}y+7$, then the correlation between X and Y is

(A) -0.5
(B) -0.75
(C) 0.6123
(D) 0.375

73. In the case of a bivariate distribution with same mean, the two regression equations are $y=ax+b$ and $x=\alpha y+\beta$. Then $\left(\frac{b}{\beta}\right)$ is

(A) $\frac{1-a}{1-\alpha}$
(B) $\frac{1+a}{1+\alpha}$
(C) $\frac{1-\alpha}{1-a}$
(D) $\frac{1+\alpha}{1+a}$

74. The lines of regression for a set of data are: $2y-x-50=0$; $3y-2x-10=0$. Then the regression estimate of y for $x=150$ and the regression estimate of x when $y=100$ is:

(A) 145, 100
(B) 100, 100
(C) 100, 145
(D) 145, 145

75. The mean deviation from the mean of the series a, $a+d$, $a+2d$, ... $a+2nd$ is:

(A) $\frac{n(n+1)d}{n+1}$
(B) $\frac{n(n+1)d}{n+2}$
(C) $\frac{n(n+1)d}{2n+1}$
(D) $\frac{n(n+1)}{2n+1}$

76. The first three moments of a variate measured from 2 are respectively 1, 16 and −40. The value of third central moment is

(A) −86 (B) 86
(C) 15 (D) 3

77. In a frequency distribution the following results were available:

mean=45, median=48; coefficient of skewness=−0.4; The value of the variance is:

(A) 25 (B) 22.5
(C) 506.25 (D) 12.5

78. The standard error of residual variance in the case of the equation of the line of regression of Y on X is :

(A) $\sigma_y(1-r^2)^{1/2}$ (B) $\sigma_y^2(1-r^2)^{1/2}$

(C) $\sigma_x^2(1-r^2)$ (D) $\sigma_y^2(1-r^2)$

79. If X is a $r.v.$ following the binomial distribution with $n=5$, $p=0.2$, then the most probable value of X taken with highest probability of X is

(A) 1 (B) 2
(C) 3 (D) 4

80. If X has a Poisson distribution with $E(X^2)=2$, then $P[X=0]$ is equal to

(A) $P[X=1]$ (B) $P[X=2]$
(C) $P[X=3]$ (D) $P[X=4]$

81. If X and Y are $r.v$'s with common variance σ^2 and correlation coefficient ρ, then the variance of $(X+Y)$ is

(A) $\sigma^2(2+\rho)$ (B) $2\sigma^2(1+\rho)$

(C) $2\sigma^2$ (D) $\sigma^2(1+2\rho)$

82. X and Y are two independent $r.v's$. X has a Poisson distribution with mean 1 and Y has the geometric distribution with $P[Y=y] = (1-p).\ p^y;\ y=0,1,2.....$ Then $P[X=Y]$ is equal to

(A) $(1-p)e^{-1}$
(B) $1-p+e^{-1}$
(C) $(1-p)e^{(p-1)}$
(D) pe^{-p}

83. Let $X_1, X_2 \ldots. X_n$ are pair-wise uncorrelated $r.v's$ with the same variance. Then correlation coefficient between X_1 and $\overline{X}$ (the mean of the $r.v's$) is

(A) 0
(B) 1
(C) $1/\sqrt{n}$
(D) 1/n

84. Let X and Y be two arbitrary $r.v's$ with $E(X)=E(Y)$ and $\text{var}(X)=\text{var}(Y)$. If $U=X+Y$ and $V=X-Y$, then U and V are

(A) independent
(B) correlated
(C) uncorrelated and thus independent
(D) uncorrelated but not independent

85. In a binomial distribution with parameters n and p, the covariance between the number of successes and the number of failures is

(A) 0
(B) 1
(C) $np(1-p)$
(D) $-np(1-p)$

86. For any binomial distribution with mean = 7 and variance = 6, the probability of success 'p' is equal to

(A) 1/49
(B) 1/42
(C) 1/7
(D) 6/7

87. If X is binomial $(1,\theta)$, then the estimator of θ^2 based on X is

(A) unbiased
(B) sufficient
(C) not unbiased
(D) not sufficient

88. Let the observed frequency in the k classes be $n_1, n_2 \ldots\ldots n_k$. If θ_i represents the probability that an observation falls in the i^{th} class, then the maximum likelihood estimator of θ_i is

(A) $\theta_i = \frac{n_i}{n}, i = 1,2\ldots\ldots k$

(B) $\theta_i = \frac{n_i}{n}, i = 1,2\ldots\ldots k-1$

(C) $\theta_i = \frac{n}{n_i}, i = 1,2\ldots\ldots k-1$

(D) $\theta_i = \frac{n}{n_i}, i = 1,2\ldots\ldots k$

89. Let $X_1, X_2\ldots\ldots X_n$ be a random sample of size n from a population whose density function is $f(x,\theta) = \frac{1}{\alpha!\,\theta^{\alpha+1}} x^{\alpha} e^{-x/\theta}, 0 < x < \infty$ where α is known. Then maximum likelihood estimate of θ is

(A) $\frac{\overline{X}}{\alpha - 1}$

(B) $\frac{\alpha + 1}{\overline{X}}$

(C) $\frac{\overline{X}}{\alpha + 1}$

(D) $\frac{\alpha - 1}{\overline{X}}$

90. Let X be a $r.v$ with $p.d.f$ $f(x) = 1, 0 \le x \le 1$
$= 0$, elsewhere.

Then the $m.g.f$ of X is

(A) $\frac{1 - e^t}{t}$ for all t

(B) $\frac{e^t - 1}{t}$ for all t

(C) $\frac{t}{1 - e^t}$ for all t

(D) $\frac{t}{e^t - 1}$ for all t

91. Given the joint density $f(x_1, x_2) = \begin{cases} \frac{k}{(1 + x_1 + x_2)^3}, & x_1 > 0, x_2 > 0 \\ 0, & \text{elsewhere} \end{cases}$, then the marginal density of X_2 is

(A) $\frac{1}{(1 + x_2)^2}$

(B) $\frac{1}{(1 + x_1)^2}$

(C) $\frac{2}{(1 + x_1)^2}$

(D) $\frac{2}{(1 + x_2)^2}$

92. Suppose that an airplane engine will fail when in flight with probability $(1-p)$ independently from engine to engine; suppose that the airplane will make a successful flight if atleast 50% of its engines remain operative, for what value of p is a four-engine plane preferable to a two-engine plane?

(A) 1/3 (B) 2/3
(C) 3/4 (D) 1/2

93. Suppose that the amount of time one spends in a bank is exponentially distributed with mean 10 minutes that $\lambda = 1/10$. The probability that a customer will spend more than 15 minutes in the bank given that he is still in the bank after 10 minutes is

(A) 0.220 (B) 0.110
(C) 0.604 (D) 0.724

94. The control chart techniques was proposed for process controls by

(A) Dodge (B) Shewart
(C) Wetherill (D) Mahalanobis

95. Let X be a normal random variable with mean zero and variance 9. If $a = P(X \geq 3)$, then $P(|X| \leq 3)$ equals

(A) a (B) $1-a$
(C) $2a$ (D) $1-2a$

96. If $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with finite variance σ^2, then an unbiased estimate of σ^2 is

(A) $\frac{\sum (x_i - \bar{x})^2}{n(n+1)}$ (B) $\frac{\sum (x_i - \bar{x})^2}{(n-1)}$
(C) $\frac{\sum (x_i - \bar{x})^2}{n}$ (D) $\frac{\sum (x_i - \bar{x})^2}{n(n-1)}$

97. If x_n and y_n are consistent estimators of parameters α and β, then (i). $(x_n + y_n)$ is consistent estimator of $(\alpha + \beta)$ (ii). $(x_n \otimes y_n)$ is consistent estimator of $(\alpha\beta)$

(A) (i) is true but not (ii)
(B) (ii) is true but not (i)
(C) both are false
(D) both are true

98. Let $X_1, X_2, \ldots, X_n$ be a random sample of n observations of X whose $p.d.f$ is $f(x,\theta) = \begin{cases} e^{-(x-\theta)}, \theta < x < \infty, -\infty < \theta < \infty \\ 0 \quad , elsewhere \end{cases}$. Then $(\bar{x} - 1)$ suggested as an estimator of θ is

(A) unbiased
(B) efficient
(C) consistent
(D) sufficient

99. Let $X_1, X_2, \ldots X_n$ are $i.i.d$ variate with $p.d.f.$ $f(x,\theta), \theta \in \Theta$. Let g be a one to one function defined on θ. If $\hat{\theta}$ is an MLE of θ, then $g(\hat{\theta})$ is an MLE of $g(\theta)$. This indicates that

(A) *MLE* is not unique
(B) *MLE* is not consistent
(C) the large sample property of *MLE*
(D) the invariance property of *MLE*

100. Let a random sample of size n be taken from a population with density $f(x,\theta) = \theta e^{-x\theta}, 0 < x < \infty$. The central confidence limits for large samples with 95% confidence coefficients are:

(A) $\theta = \left(1 + \frac{1.96}{\sqrt{n}}\right)(\bar{x})^{-1}$
(B) $\theta = \left(1 - \frac{1.96}{\sqrt{n}}\right)(\bar{x})^{-1}$
(C) $\theta = \left(1 \pm \frac{1.96}{\sqrt{n}}\right)(\bar{x})^{-1}$
(D) $\theta = \left(\frac{1.96}{\sqrt{n}}(\bar{x})^{-1}\right)$

101. Let X_1, X_2, ... Xn be a random sample of size n from a normal population with mean μ and variance σ^2. The statistic $\hat{\mu} = \frac{1}{n+1}\sum_{i=1}^{n} x_i$ is:

(A) unbiased for θ
(B) most efficient for θ
(C) sufficient for θ
(D) consistent for θ

102. Given a random sample of size n from a population whose density function $f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$, the moment estimator of σ^2 is

(A) $\hat{\sigma}^2 = \mu_2^1 - (\mu_1^1)^2$
(B) $\hat{\sigma}^2 = \mu_2^1 + \mu_1^{1^2}$
(C) $\hat{\sigma}^2 = \mu_2^1$
(D) $\hat{\sigma}^2 = \frac{\mu_2^1}{(\mu_1^1)^2}$

103. Let x_1, x_2 and x_3 be a random sample of size 3 from a population with mean value μ and variance σ^2. T_1, T_2 and T_3 are the estimators used to estimate mean value of μ where $T = x_1 + x_2 - x_3$; $T_2 = 2x_1 + 3x_3 - 4x_2$ and $T_3 = (x_1 + x_2 + x_3)/3$. The best estimator in the sense of minimum variance is

(A) T_1
(B) T_2
(C) T_3
(D) None of the above

104. Consider the probability density function $f(x,\alpha,\beta) = \left\{ \begin{array}{ll} \frac{1}{\beta-\alpha}, & \alpha \le x \le \beta \\ 0 \;, & elsewhere \end{array} \right\}$.

Denote by $y_1, y_2, \ldots. y_n$ the order statistics based on a random sample of size n. The maximum likelihood estimators of α and β are

(A) $\hat{\alpha} = y_1$; $\hat{\beta} = y_n$
(B) $\hat{\alpha} = y_n$; $\hat{\beta} = y_1$
(C) $\hat{\alpha} = \bar{x}$; $\hat{\beta} = s^2$
(D) $\hat{\alpha} = s^2$; $\hat{\beta} = \bar{x}$

105. Let $X_1, X_2, \ldots X_n$ be *i.i.d* variates from $N(\mu, \sigma^2)$ and let it be given that μ is known so that σ^2 is the only parameter to be estimated. The confidence interval of σ^2 is:

(A) $\frac{\sum(x_i-\mu)^2}{\chi^2_{n,\alpha/2}}, \frac{\sum(x_i-\mu)^2}{\chi^2_{n,1-\alpha/2}}$

(B) $\frac{\sum(x_i-\bar{x})^2}{\chi^2_{n,\alpha/2}}, \frac{\sum(x_i-\bar{x})^2}{\chi^2_{n,1-\alpha/2}}$

(C) $\frac{\sum(x_i-\mu)^2}{\chi^2_{n-1,\alpha/2}}, \frac{\sum(x_i-\mu)^2}{\chi^2_{n-1,1-\alpha/2}}$

(D) $\frac{\sum(x_i-\bar{x})^2}{\chi^2_{n-1,\alpha/2}}, \frac{\sum(x_i-\bar{x})^2}{\chi^2_{n-1,1-\alpha/2}}$

106. Let $(X_1, X_2, \ldots X_n)$ be a random sample of size n from $N(\mu, \sigma^2)$, then the minimum variance unbiased estimator of μ is (given σ^2)

(A) $\frac{\sum x_i^2}{n}$

(B) $\frac{\sum(x_i-\bar{x})^2}{n}$

(C) $\frac{\sum(x_i-\bar{\mu})^2}{(n-1)}$

(D) $\frac{\sum x_i}{n}$

107. If $X_1, X_2, \ldots X_n$ are independent random observations on a variable assuming the value 1 with probability p and the value 0 with probability $(1-p)$, then the unbiased estimator for p^2 is

(A) $\bar{x}^2$

(B) $\bar{x}(\bar{x}-1)$

(C) $\bar{x}(\bar{x}-1/n)$

(D) $\bar{x}(\bar{x}+\frac{\bar{x}-1}{n-1})$

108. In sampling from the population with *p.d.f* $f(x,\theta)=\frac{1}{\pi[1+(x-\theta)^2]}, -\infty<x<\infty; -\infty<\theta<\infty$ the Cramer-Rao lower bound for the variance of an unbiased estimator of θ is

(A) $n/2$

(B) $2/n$

(C) $2/n^2$

(D) $n^2/2$

109. Let $\{X_n\}$ be pairwise independent and identically distributed random variables with finite mean m. The statement $\frac{S_n}{n} \rightarrow m$ in probability leads to

(A) Cramer –Rao inequality
(B) Strong law of large numbers
(C) Weak law of large numbers
(D) Central limit theorem

110. If $X \geq 1$ is the critical region for testing $\theta = 2$ against the alternative $\theta = 1$, on the basis of a single observation from the population $f(x,\theta) = \theta e^{-\theta x}, 0 < x < \infty$. The value of power of the test is:

(A) $1/e$
(B) $1/e^2$
(C) $(e-1)/e$
(D) $(e+1)/e$

111. Given a density function $f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta \\ 0 & \text{elsewhere} \end{cases}$. If we are testing the hypothesis $H_0 : \theta=1$ against $H_1 : \theta=2$ by means of a single observation of x with critical region $1 \leq x \leq 1.5$, then the power of the test is

(A) 3/4
(B) 1/4
(C) 1/2
(D) 3/8

112. The critical region of a test of hypothesis is based on the

(A) probability of a correct decision
(B) probability of not making a correct decision
(C) probability of committing type-I error
(D) probability of committing type-II error

113. In a sampling method, if the same sample unit is obtained more than one time then the method of sampling is

(A) SRSWOR
(B) systematic sampling
(C) SRSWR
(D) None of the above

114. The following test procedure is used to test the hypothesis of unbiasedness of a coin against alternative that the probability of head is 2/3. The coin is tossed four times and the hypothesis rejected if all four result in heads. The correct value of (α, β) are

(A) (1/16, 16/81) (B) (1/16, 65/81)
(C) (1/81, 16/81) (D) (1/81, 15/16)

115. Which of the following represent a composite hypothesis in a normal population with mean μ and variance σ^2, using the codes given below?

1) $\mu=1$ 2) $\sigma^2=1$ 3) $\mu=\sigma^2$

Select the correct answer.

(A) 1 and 2 are correct (B) 2 and 3 are correct
(C) 1,2 and 3 are correct (D) None of the above

116. Area of the critical region depends on

(A) size of type I error (B) size of type II error
(C) power of the test (D) the number of observations

117. A critical region, in which probability of rejecting H_0 when it is not true is less than of rejecting it when it is true, is said to be

(A) biased (B) unbiased
(C) uniformly most powerful (D) uniformly most powerful unbiased

118. The term which correctly defines "test function" for a given sample observation $x = (x_1, x_2 .. x_n)$ is

(A) conditional probability of rejecting the null hypothesis
(B) probability of rejecting the null hypothesis
(C) conditional probability of accepting the null hypothesis
(D) probability of accepting the null hypothesis

119. Let 'p' be the probability of getting a head in a single toss of a coin. To test $H_0 : p = 1/2$ against $H_1 : p = 3/4$, a coin is tossed 5 times. H_0 is rejected if more than 3 heads are obtained. The probability of type I error is

(A) 24/128 (B) 51/128
(C) 104/128 (D) None of the above

120. A statistical concept wholly based on observations of a characteristic in a finite sample is known as

(A) statistic
(B) parameter
(C) degrees of freedom
(D) level of significance

121. The efficiency of SRSWOR with respect to SRSWR is given by

(A) $\frac{N-n}{N-1}$
(B) $\frac{N-n}{n-1}$
(C) $\frac{n-1}{N-1}$
(D) $\frac{N-1}{N-n}$

122. The number of samples of size 5 that may be drawn from a population consisting of 20 distinct unit is

(A) 100
(B) 2400
(C) 15,502
(D) 15,504

123. The variance of a population of size 80 is 2. A random sample of size 32 is drawn according to SRSWR. The standard error of the unbiased estimate of the population total will be

(A) 1/4
(B) 1/16
(C) 5
(D) 20

124. If a faulty measurement is recorded on a certain sample unit due to evasive response furnished by the unit, then it is a case of

(A) non-sampling error
(B) sampling error
(C) neither sampling nor non-sampling error
(D) both sampling and non-sampling error

125. A whole number x is chosen at random between 1 and 5 inclusive. The number y is then chosen at random between 1 and x. The expected value of y is

(A) 2
(B) 1
(C) 5/2
(D) 3

126. Which one of the following is not a moment generating function?

(A) $\frac{t}{1+t^2}$ (B) 1

(C) $\frac{t}{3-2e^t}$ (D) e^{10t+2t^2}

127. Let (X, Y) be jointly distributed with density function $f(x,y)=\begin{cases} e^{-x-y}, & 0<x<\infty,\ 0<y<\infty \\ 0, & otherwise \end{cases}$. Then the moment generating function is

(A) $\frac{1}{(1+t_1)(1-t_2)}$ (B) $\frac{1}{(1-t_1)(1+t_2)}$

(C) $\frac{1}{(1-t_1)(1-t_2)}$ (D) $\frac{1}{(1+t_2)}$

128. Two random variables X and Y have the following joint probability density function $f(x,y)=\begin{cases} 2-x-y, & 0\le x\le 1;\ 0\le y\le 1 \\ 0, & otherwise \end{cases}$. The covariance between X and Y is

(A) 1/144 (B) −1/144

(C) 1/16 (D) 5/12

129. Given the joint density function $f(x_1,x_2)=\begin{cases} \frac{k}{(1+x_1+x_2)^3}, & x_1>0,\ x_2>0 \\ 0, & otherwise \end{cases}$.

The marginal density of X_1 is

(A) $\frac{1}{(1+x_1)^2}$ (B) $\frac{1}{(1+x_2)^2}$

(C) $\frac{2}{(1+x_1)^2}$ (D) $\frac{2}{(1+x_2)^2}$

130. Given the probability function.

X	:	0	1	2	3
$p(x)$	:	0.1	0.3	0.5	0.1

The variance of the random variable X is

(A) 1.6
(B) 16.24
(C) 15.8
(D) 6.4

131. A, B and C are three arbitrary events. Identify the correct expression for 'only B occurs'

(A) $\overline{A} \cap B \cap C$
(B) $\overline{A} \cap B \cap \overline{C}$
(C) $A \cap B \cap C$
(D) $A \cap B \cap \overline{C}$

132. In a very hotly fought battle, the probability of a combatant losing an eye was 70%, an ear 75%, an arm 80% and a leg 85%. The probability of a combatant losing all the four (in percentage) was not less than

(A) 8
(B) 10
(C) 12
(D) 11

133. Let X_1 and X_2 have the joint *p.d.f.* $f(x_1, x_2) = \begin{Bmatrix} 2, & 0 < x_1 < x_2 < 1 \\ 0, & otherwise \end{Bmatrix}$. The conditional variance of X_1, given $X_2 = x_2$ is

(A) $\frac{x_2}{2}, 0 < x_2 < 1$
(B) $\frac{x_2}{4}, 0 < x_2 < 1/2$
(C) $\frac{x_2^2}{12}, 0 < x_2 < 1$
(D) $\frac{x_2^2}{8}, 0 < x_2 < 1$

134. For any two events A and B, $P(A \cap B) - P(A).P(B)$ is equal to

(A) $P(A)P(B') - P(A \cap B')$
(B) $P(A \cap B') - P(A)P(B')$
(C) $P(A \cap B') - P(A').P(B')$
(D) $P(A' \cap B) - P(A)P(B')$

135. A_1 and A_2 are independent events. Consider the statements (1) A_1 and $\overline{A_2}$ are independent (2) $\overline{A_1}$ and $\overline{A_2}$ are independent. Then

(A) (1) is true and (2) is false
(B) (1) is false and (2) is true
(C) (1) and (2) are both true
(D) (1) and (2) are both false

136. Let X be a continuous *r.v.* with *p.d.f.* $f(x) = \begin{cases} ax, & 0 < x < 1 \\ a, & 1 < x < 2 \\ -ax + 3a, & 2 < x < 3 \\ 0, & elsewhere \end{cases}$. This implies that the value of a is

(A) 0.5
(B) 2
(C) 1.5
(D) 1.0

137. Let $f(x) = \begin{cases} k e^{-x} x^{-1/2}, & 0 < x < \infty \\ 0, & elsewhere \end{cases}$. The value of k for $f(x)$ to be a *p.d.f.*, is

(A) π
(B) $\frac{2}{\pi}$
(C) $\frac{2}{\sqrt{\pi}}$
(D) $\frac{1}{\sqrt{\pi}}$

138. If $\{X_n\}$ is a sequence of independent and identically distributed *r.v's* with finite mean and variance, then $\{X_n\}$ satisfies

(A) WLLN but not necessarily CLT
(B) CLT but not necessarily WLLN
(C) neither WLLN nor CLT without further conditions
(D) both WLLN as well as CLT without any other conditions

139. Let X_i, i=1,2,3…n be independent Bernoulli random variables with $P(X_i=1) = p$; $P(X_i=0)=1-p$. Let $S_n=X_1+X_2+...X_n$. If $E(S_n)=42$ and $V(S_n) = 6$, then the values of n and p will be

(A) $n=49, p=1/7$
(B) $n=49, p=6/7$
(C) $n=84, p=1/2$
(D) $n=126, p=1/3$

140. A lock is to be opened and its key is in a bunch of five keys, with none of the other four fitting the lock. If one tries the keys at random and without repetition, then the probability that the door opens in the third attempt is

(A) 1/5
(B) 3/5
(C) $(5/3)(4/5)^3(1/5)^2$
(D) $(5/2)(4/5)^2(1/5)^3$

141. If X is uniformly distributed over (0, 10), the probability $1 < X < 6$ is

(A) 1/2
(B) 3/10
(C) 4/7
(D) 5/7

142. The failure rate of a new process is estimated at 75%. How many times should the process be run to give an 80% of at least two successes?

(A) 10
(B) 11
(C) 9
(D) 12

143. If $X \sim U(-b,b)$, the value of b such that $P(|x| > 2) = 3/4$ is

(A) 6
(B) 8
(C) 5
(D) 2

144. A random variable 'X' has Poisson distribution such that $P(2) = 9P(4) + 90P(6)$, then $E(X)$ is equal to

(A) 2
(B) 2.5
(C) 1
(D) 3

145. Given the bivariate normal distribution

$$f(x,y) = \left(2\pi\sqrt{3}\right)^{-1} \exp\left\{-\left((2x-y)^2 + 2xy\right)/6\right\} - \infty < x, y < \infty.$$

The correlation between X and Y is

(A) +1
(B) −1
(C) ±1
(D) $\sqrt{3}$

146. If X is a Poisson variate with mean 'm', then the expectation of e^{-kx} where k is a constant is

(A) $\exp(m(1-e^{-k}))$
(B) $\exp(-m(e^{-k}-1))$
(C) $\exp(m(1-e^{k}))$
(D) $\exp(-m(1-e^{-k}))$

147. Let X_1 and X_2 are the standard normal variates with correlation coefficient ρ between them. Then the correlation coefficient between X_1^2 and X_2^2 is

(A) $+\rho$
(B) $-\rho$
(C) ρ^2
(D) 0

148. For a certain $N(\mu, \sigma^2)$, the first moment about 10 is 40 and the fourth moment about 50 is 48. The variance of $N(\mu, \sigma^2)$ is

(A) 4
(B) 2
(C) 50
(D) 40

149. Poisson distribution is related with events which are

(A) often occurring
(B) frequent
(C) of rare occurrence
(D) None of the above

150. In design of experiments,

(A) randomisation and replication assures the validity of estimate for error
(B) randomisation and local control reduce the experimental error
(C) local control and replication assure the validity of the estimate of error
(D) randomisation is not always required

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