STATISTICS
(FINAL)

1. Let X_1, X_2, X_3 be random observations from a population with mean M. Some estimators of M are suggested below

   1. $T_1 = (X_1 - 2X_2)$
   2. $T_2 = (2X_2 - X_3)$
   3. $T_3 = \frac{(X_1 + X_2 + X_3)}{3}$
   4. $T_4 = \frac{(X_1 + 3X_2 + X_3)}{5}$

   Which of the above estimators are unbiased?

   (A) T_1 and T_2 only
   (B) T_3 and T_4 only
   (C) T_2, T_3 and T_4 only
   (D) T_1, T_2, T_3 and T_4

2. Let $X_1, X_2, \ldots, X_n$ be a sample from $U(\theta, \theta+1)$, where $n>1$. Then consider the following three estimators of $\theta, \theta \in R$

$$T_1 = X_{(1)}; T_2 = \frac{X_{(1)} + X_{(n)}}{2}; T_3 = \frac{X_{(1)} + X_{(n)}}{2} - \frac{1}{2}$$

   Then which of the above estimators is/are maximum likelihood estimator of θ?

   (A) T_1 and T_3 only
   (B) T_1 and T_2 only
   (C) T_3 only
   (D) T_1, T_2 and T_3

3. Suppose X is a random variable with finite variance. For $0<\theta<1$ and $n>3$, let $X_1 = X,\ X_2 = \theta X_1,\ X_3 = \theta X_2, ..., X_n = \theta X_{n-1}$. Then $corr\left(X_1, X_n\right)$ is

(A) 1

(B) -1

(C) θ^{n-2}

(D) θ^{n-3}

4. Let $X_1, X_2, \ldots, X_n (n>1)$ be a sample from exp(1). Then the distribution of $2n\bar{X}$ is

(A) $\exp\left(\frac{1}{2}\right)$

(B) $\exp(2n)$

(C) χ^2_n

(D) χ^2_{2n}

5. Let X be a random variable having probability density function

$$f(x; x_0, \alpha) = \begin{cases} \frac{\alpha x_0^{\alpha}}{x^{\alpha+1}}, & \text{if } x > x_0 \\ 0, & \text{if } x \le x_0 \end{cases}$$

Where $\alpha,\ x_0 > 0$. If $Y = \log\left(\frac{x}{x_0}\right)$ then $P(Y>3)$ is

(A) $\exp(-3\alpha x_0)$

(B) $1-\exp(-3\alpha x_0)$

(C) $1-\exp(-3\alpha)$

(D) $e^{-3\alpha}$

6. Let the random variable X~ binomial (3, θ), $0<\theta<1$. A test of hypothesis $H_0 : \theta = \frac{3}{4}$ against $H_1 : \theta = \frac{1}{4}$ reject H_0 if $X \leq 1$. Then the test has

(A) size = $\frac{5}{32}$, power = $\frac{27}{32}$

(B) size = $\frac{5}{32}$, power = $\frac{18}{32}$

(C) size = $\frac{15}{32}$, power = $\frac{27}{32}$

(D) size = $\frac{1}{32}$, power = $\frac{31}{32}$

7. Let $X_1, X_2, \ldots, X_{20}$ be a random sample from a distribution having the variance 5. Let $\bar{X}$ denote the mean of the given sample and $S = \sum_{i=1}^{20}\left(X_i - \bar{X}\right)^2$. Then the value of $E(S)$ is

(A) 5
(B) 95
(C) 0.25
(D) 100

8. Four identical fair dice are thrown independently. Let S denote the number of dice showing odd number on their upper faces. Then the variance of the random variable S is

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 1

(D) 2

9. Let the joint probability density function of X and Y be

$$f(x,y)=\begin{cases}\exp(-x) \text{ if } 0\le y\le x<\infty\\ 0 \qquad\qquad \text{otherwise}\end{cases}$$

Then $E(X)$ is

(A) 0.5
(B) 1
(C) 2
(D) 6

10. Let X be a continuous random variable with *pdf* symmetric about 0, that is $f(-x)=f(x),\forall x\in R$. If $V(X)<\infty$, then which of the following statement is TRUE?

(A) $E\left(|X|\right)=E(X)$
(B) $V\left(|X|\right)=V(X)$
(C) $V\left(|X|\right)<V(X)$
(D) $V\left(|X|\right)>V(X)$

11. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be *iid* $N(0, 1)$ random variables. Then

$W=\dfrac{n\left(\sum_{i=1}^{m}X_i\right)^2}{m\left(\sum_{j=1}^{n}Y_i\right)^2}$ has

(A) χ^2_{m+n} distribution
(B) t_n distribution
(C) $F_{m,n}$ distribution
(D) $F_{1,1}$ distribution

12. X_1, X_2, X_3 and, X_4 be independent random variables. Then which of the following pairs of random variables are independent?

(A) $\left(X_1 + X_2,\ X_2 + X_3\right)$

(B) $\left(X_1,\ X_1 + X_3\right)$

(C) $\left(X_1 + X_2,\ X_3\right)$

(D) $\left(X_2,\ X_1 + X_2 + X_3\right)$

13. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu,16)$ population. If a 95% confidence interval for μ is $\left[\bar{X} - 0.98, \bar{X} + 0.98\right]$, then the value of n is

(A) 4
(B) 16
(C) 32
(D) 64

14. Let the random variable X follows binomial distribution with parameters n and p, then $\operatorname{cov}\left(\frac{X}{n}, \frac{n-X}{n}\right)$ equals to

(A) npq

(B) $-pq$

(C) $\frac{pq}{n}$

(D) $-\frac{pq}{n}$

15. Let X follows the geometric distribution with parameter p, then consider the following statements for non-negative integers a, b:

1. $P(X \geq a+b \mid X \geq a) = P(X \geq b)$
2. $P(X = a+b \mid X \geq a) = P(X = b)$

Which of the above statements is/are TRUE?

(A) 1 only
(B) 2 only
(C) 1 and 2
(D) neither 1 nor 2

16. Let the random variable $X \sim \chi^2_n$ .Then the distribution of $U = \frac{X-n}{\sqrt{2n}}$, for large n is

(A) $N(0,1)$

(B) χ^2_{n-1}

(C) χ^2_{n-2}

(D) $N\left(1, \frac{1}{2}\right)$

17. Let X_1, X_2be a random sample from $N(0, 1)$ and Y_1, Y_2 another sample from a different distribution $N(1, 1)$. Then the distribution of $\overline{X} + \overline{Y}$ is

(A) $N(1,2)$

(B) $N\left(1, \frac{1}{2}\right)$

(C) $N(1,1)$

(D) $N(1, \sqrt{2})$

18. Let X_1, X_2, …., $X_n$be iid exp(1) random variables and $S_n = \sum_{i=1}^{n} X_i$. Using the central limit theorem, the value of $\lim_{n\to\infty} P(S_n > n)$ is

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) 1

19. Let $X_1, X_2, \ldots, X_8$ be iid $N(0, \sigma^2)$ random variables. Further, let $U = X_1 + X_2$ and $V = \sum_{i=1}^{8} X_i$. The correlation coefficient between U and V is

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) $\frac{1}{2}$

20. Two coins with probability of head u and v, respectively, are tossed independently. If P (both coins show up tails) = P (both coins show up head), then $u+v$ equals

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

21. The observed value of mean of a random sample from $N(\theta, 1)$ distribution is 2.3. If the parameter space is $\Theta = \{0, 1, 2, 3\}$, then the maximum likelihood estimate of θ is

(A) 1
(B) 2
(C) 2.3
(D) 3

22. Let X be any random variable with mean μ and variance 9. Then the smallest value of m such that $P\left(\left|X-\mu\right|<m\right)\geq 0.99$ is

(A) 90

(B) $\sqrt{90}$

(C) $\sqrt{\frac{100}{11}}$

(D) 30

23. The skewness of binomial distribution $B(n, p)$ becomes 0 when

(A) $p>\frac{1}{2}$

(B) $p=\frac{1}{2}$

(C) $p<\frac{1}{2}$

(D) $p<q$

24. Let t follows Student's t distribution with n degrees of freedom, then t^2 follows

(A) $\chi^2_{(1)}$

(B) $\chi^2_{(n)}$

(C) $F(1,n)$

(D) $F(2,n)$

25. Let $\bar{X}$ be the mean of a random sample of size n drawn from $N(\theta,4)$. Then which of the following is NOT a consistent estimator of θ?

(A) $\frac{1}{n}+n\bar{X}$

(B) $\frac{n\bar{X}+1}{n+5}$

(C) $\frac{1}{n(n+1)}+\frac{n\bar{X}}{n+1}$

(D) $\bar{X}+\frac{1}{n}$

26. Let X_1, X_2 be a random sample from a $N(0,\theta)$ distribution where $\theta>0$. Then the value of K, for which the interval $\left(0,\frac{X_1^2+X_2^2}{K}\right)$ is 95% confidence interval of θ, equals

(A) $-\log_e(0.95)$

(B) $-2\log_e(0.95)$

(C) $-\frac{1}{2}\log_e(0.95)$

(D) 2

27. Let the random vector (X,Y) have a bivariate normal distribution with parameter $\mu_x=0,\ \mu_y=0, \sigma_x^2=1,\ \sigma_y^2=1$ and $corr(X,Y)=\rho$, then $V(X-Y)$ is equal to

(A) $2\rho^2$

(B) $2(1+\rho)$

(C) $2(1-\rho)$

(D) $2(1-\rho^2)$

28. Two regression lines are given as follows

$$8X - 10Y + 66 = 0 \text{ and } 40X - 18Y = 214$$

Then means of X and Y, respectively are

(A) $\left(\frac{4}{5}, \frac{9}{20}\right)$

(B) (8,10)

(C) (40,18)

(D) (13,17)

29. From the frequency distribution with open-end class interval at the end, we can calculate

1. Mean
2. Median
3. Mode

Choose your answer from the following

(A) 1 and 2
(B) 2 and 3
(C) 1 and 3
(D) 1, 2 and 3

30. If the regression line of Y on X is $Y + 0.8X = 25$ and the standard deviations of X and Y, respectively are 3 and 8. Then the value of correlation coefficient r is

(A) –0.3
(B) –0.4
(C) 0.3
(D) 0.4

31. If X and Y are two independent Poisson variates such that $X \sim P(1)$ and $Y \sim P(2)$, the $P(X+Y<3)$ is

(A) e^{-3}

(B) $3e^{-3}$

(C) $4e^{-3}$

(D) $8.5e^{-3}$

32. Which one of the following is/are true for independent random variables X and Y?

(i) $X \sim B(5,0.2)$, $Y \sim B(5, 0.4)$ then $X + Y \sim B(10, 0.6)$
(ii) $X \sim P(5)$, $Y \sim P(7)$, then $X+Y \sim P(12)$

Choose your answer from the following:

(A) Both (i) and (ii) are true
(B) (i) is true but (ii) is false
(C) (i) is false but (ii) is true
(D) Both (i) and (ii) are false

33. Let $X \sim N(\mu, \sigma^2)$ both μ and σ^2 be unknown. Let μ_0 be a known constant. Consider the null hypothesis $H_0 : \mu \leq \mu_0,\ \sigma^2 > 0$ and the alternative hypothesis $H_1 : \mu > \mu_0,\ \sigma^2 > 0$, then

(A) both null and alternative hypotheses are simple
(B) both null and alternative hypotheses are composite
(C) null is simple and alternative hypothesis is composite
(D) null is composite and alternative hypothesis is simple

34. Let E, F and G be three events such that the events E and F are mutually exclusive, $P(E \cup F) = 1$, $P(E \cap G) = \frac{1}{4}$ and $P(G) = \frac{7}{12}$. Then $P(F \cap G)$ equals

(A) $\frac{1}{12}$

(B) $\frac{1}{4}$

(C) $\frac{5}{12}$

(D) $\frac{1}{3}$

35. Let $P = \begin{pmatrix} 1 & 2 & 0 & 2 \\ -1 & -2 & 1 & 1 \\ 1 & 2 & -3 & -7 \\ 1 & 2 & -2 & -4 \end{pmatrix}$. Then rank of P equals

(A) 4
(B) 3
(C) 2
(D) 1

36. Let $P = \begin{pmatrix} 1 & 0 & 1+x & 1+x \\ 0 & 1 & 1 & 1 \\ 1 & 1+x & 0 & 1+x \\ 1 & 1+x & 1+x & 0 \end{pmatrix}$. Then the determinant of matrix P is

(A) $3(x+1)^3$

(B) $3(x+1)^2$

(C) $3(x+1)$
(D) $(x+1)(2x+3)$

37. The Matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$

(A) is an elementary matrix
(B) can be written as a product of elementary matrices
(C) does NOT have linearly independent eigenvectors
(D) is a nilpotent matrix

38. In the Neyman-Pearson set up,if α and β denote probabilities of Type I and Type II errors, then we

(A) minimize both α and β
(B) minimize α for fixed β
(C) fix α and maximize β
(D) fix α and maximize $(1-\beta)$

39. The technique of analysis of variance was developed by

(A) R. A. Fisher
(B) Irvin Fisher
(C) Neyman
(D) C.R. Rao

40. Which one of the following is not a basic principle of design of experiment?

(A) Randomization
(B) Local control
(C) Test of significance
(D) Replication

41. The correct relationship between AM, GM and HM is

(A) AM ≥ GM ≥ HM
(B) AM ≤ GM ≤ HM
(C) GM ≤ HM ≤ AM
(D) GM ≤ AM ≤ HM

42. In a population of size 50, a systematic sample of size 5 is drawn. If a unit selected in the sample is 17, then the other units of the sample will be,

(A) (7,15,25,35)
(B) (7, 27, 37, 47)
(C) (7, 20, 18, 27)
(D) (17, 18, 25, 27)

43. If T is an estimator of parameter θ, then

(A) $MSE(T) = V(T)$
(B) $V(T) = MSE(T) + Bias(T)$
(C) $MSE(T) = V(T) + (Bias(T))^2$
(D) $V(T) = MSE(T) + (Bias(T))^2$

44. Power of the test is the probability of

(A) Rejecting H_0 when H_0 is true

(B) Rejecting H_1 when H_0 is true

(C) Rejecting H_0 when H_1 is true

(D) Rejecting H_1 when H_1 is true

45. The infinite sum of $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+...$ is equal to which of the following?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

46. Given the infinite series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-...$., which of the following is true?

(A) The series converges
(B) The series diverges to infinity
(C) The series oscillates
(D) The series does not converge

47. For the infinite sum of $\frac{1}{(n!)}$ from $n = 1$ to infinity, where $n!$ denotes the factorial of n, which of the following is **NOT** true?

(A) The infinite sum converges to 0
(B) The partial sum sequence of the infinite sum converges
(C) The n-th term of the infinite sum converges to 0 as n tends to infinity
(D) The infinite sum converges to $e - 1$

48. Given a real valued function f of a real variable, which of the following need **NOT** be true?

(A) If f is continuous at a point x, then f is differentiable at the point x
(B) If f is differentiable at a point x, then f is continuous at the point x
(C) If f is continuous at a point x, then f need not be differentiable at that point x

(D) The domain of f can contain points at which f is continuous but not differentiable

49. For a real valued function f of a real variable, if $f(A)$ is the image set of A containing all $f(x)$, x in A, which of the following is the image set of the open interval $(-1,1)$, given that $f(x) = x^2$?

(A) The open interval $(-1, 1)$
(B) The open interval $(-1, 0)$
(C) The open interval $(0,1)$
(D) The union of the open interval $(0,1)$ and the singleton $\{0\}$

50. If $f(x) = \log x,\ x > 0$, and $g(x) = \exp(x),\ x$ any real number, log to base e and exp is the exponential function, what is $f\left(g(0)\right) + g\left(f(1)\right)$?

(A) 0
(B) 1
(C) 2
(D) e

51. If $f(x) = x + |x|,\ -1 < x < 1,$ which of the following is **NOT** true?

(A) f is a continuous function in $(-1,1)$
(B) f is a differentiable function in $(-1,1)$
(C) f is a differentiable function in $(-1,1)$ except at 0
(D) f is a nonnegative function in $(-1,1)$

52. If $f(x) = \exp(x),\ x < 0,$ where exp(.) is the exponential function, what is integral of $f(x)dx$ over – infinity to 0?

(A) 0
(B) Integral does not exist
(C) – Infinity
(D) 1

53. If f is a strictly increasing and continuous function, which of the following is true?

(A) f has a unique inverse function

(B) Inverse of f is strictly decreasing
(C) Inverse of f need not be continuous
(D) Inverse of f is not unique

54. If $f(x) =$ modulus of $(\sin x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, what is the minimum value of $f(x)$ in its domain?

(A) 0
(B) -1
(C) 1
(D) $\frac{1}{2}$

55. If $f(x)$ = minus modulus of (x), $-1 < x < 1$, what is integral of $f(x)dx$ from $x = -1$ to $x = 1$?

(A) 1
(B) $\frac{1}{2}$
(C) $-\frac{1}{2}$
(D) -1

56. If $f(x) = \ln(x)$, $x > 0$, where ln is the natural logarithm to base e, what is the integral of $f(x)dx$ over 1 to e ?

(A) 0
(B) 1
(C) e
(D) -1

57. What is $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$?

(A) e
(B) 0
(C) 1
(D) $\frac{1}{e}$

58. What is $\lim_{n\to\infty} \frac{1-n}{1+n}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) −1

59. What is $\lim_{n\to\infty} \left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)$?

(A) Limit is not finite
(B) 1
(C) e
(D) 0

60. Which of the following is **NOT** true with reference to the series $\left(1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+...\right)$?

(A) The series converges
(B) The series diverges to infinity
(C) The n-th term of the series converges to 0 as n tends to infinity
(D) The series neither oscillates nor diverges

61. What is the value $\int_{-\infty}^{0} \frac{e^{-\left(\frac{x^2}{2}\right)}}{\sqrt{2\pi}} dx$?

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) Infinity

62. If A is a real, square matrix, which of the following statements need **NOT** be true?

(A) Square of A is defined

(B) $A^T A$ is symmetric matrix
(C) AA^T is symmetric matrix
(D) Rank of A is equal to trace of A

63. Given that real, square matrix A has determinant equal to 0, which of the following is true?

(A) A is a singular matrix
(B) A is a non-singular matrix
(C) Inverse of A exists
(D) A has full row rank

64. Given that A is a real and orthogonal matrix, which of the following is **NOT**true?

(A) Inverse of A is equal to transpose of A
(B) Determinant of A is not equal to 0
(C) A has full row rank
(D) Determinant of A is equal to 0

65. Which of the following should be satisfied for existence of inverse of a real matrix A?

(A) A should have non-zero rows
(B) A should have non-zero columns and non-zero rows
(C) Determinant of A should be non-zero
(D) Determinant of A should be zero

66. Which of the following is an idempotent matrix?

(A) Any square matrix
(B) Any symmetric matrix
(C) Any identity matrix
(D) Any singular matrix

67. Given that R = rank of a matrix A, which of the following is **NOT** true?

(A) R remains unchanged if rows of A are interchanged
(B) R remains unchanged if any column of A is multiplied by a non-zero scalar

(C) R remains unchanged if any row of A is replaced by zeroes
(D) R remains unchanged if the rows and columns of A are interchanged

68. Given a vector space V, which of the following is **NOT** true?

(A) V has a zero vector
(B) V is closed under vector addition
(C) V is closed under scalar multiplication
(D) V is closed under vector multiplication

69. For two events A and B with $P(A)=\frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(A \text{ given } B) = \frac{1}{3}$, which of the following gives the value of $P(A \text{ union } B)$?

(A) $\frac{5}{12}$
(B) 1
(C) $\frac{2}{3}$
(D) 0

70. For two independent events A and B, which of the following need **NOT** be true?

(A) $P(A) = P(A \text{ given } B)$
(B) $P(\text{ (complement of } A) \text{ intersection } B) = P(B).P(\text{complement of } A)$
(C) $P(A)$ is less than or equal to $P(B \text{ given } A)$
(D) $P(\text{ (complement of } A) \text{ intersection (complement of } B)\text{ }) = P(\text{complement of } A).P(\text{complement of } B)$.

71. Given that every outcome in event A is in event B, which of the following is **NOT** true?

(A) $P(\text{complement of } A)$ is greater than or equal to $P(\text{complement of } B)$
(B) $P(\text{intersection of } A \text{ and } B) = P(A)$
(C) $P(\text{intersection of } A \text{ and } B) = P(B)$
(D) $P(A)$ is less than or equal to $P(B)$

72. Given that A and B are any two independent events, which of the following is true?

(A) P(union of A and B) = $P(A) + P(B)$
(B) P(union of A and B) is less than or equal to $P(A) + P(B)$
(C) P(union of A and B) is greater than or equal to $P(A) + P(B)$
(D) P(intersection of A and B) = 0

73. Given the set of numbers {0.1, 0.3, 0.4, 0.2}, which of the following statements is true?

(A) The set can be the probability mass function of a random variable assuming 4 values
(B) The set cannot be the probability mass function of any random variable
(C) The numbers in the set cannot be assumed to be probabilities
(D) The numbers in the set are probabilities of a continuous real valued random variable realizing four values

74. Given that A is the event of observing first a Head and then a Tail in two independent throws of a fair coin, what is $P(A)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{3}$

75. What is the probability of observing Tails for the first time in the fifth throw of a fair coin which is thrown independently repeatedly?

(A) $\frac{1}{2}$

(B) $\frac{1}{10}$

(C) $\frac{1}{32}$

(D) $\frac{1}{64}$

76. What is the probability that the sum of numbers facing up in two fair dice, thrown simultaneously, is an odd number, when each dice has six faces with the numbers 1 to 6 on its six faces?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{12}$

77. Which of the following is true given that X is a random variable with $P(X = 0) = \frac{1}{3} = 1 - P(X=1)$?

(A) Expected value of X is $\frac{2}{3}$ Variance of X is $\frac{1}{9}$

(B) Expected value of X is $\frac{2}{3}$ Variance of X is $\frac{2}{9}$

(C) Expected value of X is $\frac{1}{3}$ Variance of X is $\frac{1}{9}$

(D) Expected value of X is $\frac{1}{3}$ Variance of X is $\frac{2}{9}$

78. Given that X is a random variable with expected value equal to 0 and variance equal to 1, what is expected value of square of X?

(A) 0
(B) 1
(C) 2
(D) 4

79. What is the standard deviation of the random variable X given that $P(X = k)$ is equal to $\frac{1}{3}$ times $\left(\frac{2}{3}\right)$ to the power of k, $k = 0, 1, 2, \ldots$?

(A) $\sqrt{2}$
(B) 2
(C) $\sqrt{6}$
(D) $\sqrt{8}$

80. Given that random variable X has Binomial distribution with parameters $n = 10$ and $p = \frac{1}{2}$, what is the coefficient of variation of X?

(A) $\frac{1}{10}$
(B) $\frac{1}{100}$
(C) $\frac{1}{50}$
(D) $\frac{1}{\sqrt{10}}$

81. Given that X and Y are independent Poisson random variables with mean 1, which of the following is **NOT** true?

(A) $X + Y$ has Poisson distribution with mean 2
(B) $X + Y$ has Poisson distribution with variance 2
(C) $X + Y$ has Poisson distribution with variance 1
(D) $X + Y$ has Poisson distribution with mean 2 and square of coefficient of variation equal to one half

82. Given that X and Y are independent standard normal random variables, which of the following is true?

(A) Expected value of $(2X - 4Y)$ is 0 and Variance of $(2X - 4Y)$ is 20
(B) Expected value of $(2X - 4Y)$ is –2 and Variance of $(2X - 4Y)$ is 10
(C) Expected value of $(2X - 4Y)$ is –2 and Variance of $(2X - 4Y)$ is 20
(D) Expected value of $(2X - 4Y)$ is 0 and Variance of $(2X - 4Y)$ is 10

83. Given that X is a standard exponential random variable, what is the coefficient of variation of $(X+1)$?

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) 0
(D) 1

84. Which of the following is true if X is the uniform random variable over the interval (0, 1)?

(A) Expected value of X is 1
(B) Variance of X is equal to $\frac{1}{12}$
(C) Variance of X is equal to $\frac{1}{10}$
(D) Variance of X is equal to $\frac{1}{2}$

85. Given that X has uniform distribution over (0, 1), which of the following is true, where ln denotes the natural logarithm to base e?

(A) $-\ln(X)$ has standard exponential distribution
(B) $-\ln(X)$ has standard normal distribution
(C) $-\ln(X)$ has chi-square distribution
(D) $-\ln(X)$ has gamma distribution

86. Given that $X_1, X_2, \ldots, X_9$ are independent random variables, all having standard normal distribution, what is the distribution of their sum (X_1 + ...+ X_9)?

(A) Standard normal distribution
(B) Normal distribution with mean 0 and variance 10
(C) Normal distribution with mean 9 and variance 9
(D) Normal distribution with mean 0 and variance 9

87. Which of the following is the moment generating function of the standard normal distribution?

(A) *e* raised to the power of (*t* divided by 2), *t* real.
(B) *e* raised to the power of ((square of *t*) divided by 2), *t* real
(C) Reciprocal of square root of pi times *e* raised to the power of ((square of *t*) divided by 2), *t* real
(D) Reciprocal of square root of (twice pi) times *e* raised to the power of ((square of *t*) divided by 2), *t* real

88. Given that $P(s)$, s in the closed interval [0, 1], is the probability generating function of a random variable X taking values 0, 1, 2, …, which of the following is true?

(A) $P(0)$ = Probability that X takes the value 0 and $P(1) = 1$
(B) $P(0) = 0$ and $P(1)$ is the Probability that X takes the value 1
(C) $P(0)$ = Probability that X takes the value 0 and $P(1) = 0$
(D) $P(0) = 1$ and $P(1)$ = Probability that X takes the value 1

89. Given a completely randomized design with total number of observations equal to 45 and total number of treatments equal to 10, which of the following gives the degrees of freedom for the error?

(A) 34
(B) 35
(C) 36
(D) 450

90. Which of the following is not related to design of statistical experiments?

(A) Design matrix
(B) Least squares estimates

(C) Global control
(D) Replication

91. Given a completely randomized design model, which of the following is true?

(A) The yield of interest depends on one source of heterogeneity
(B) The yield of interest depends on two sources of heterogeneity
(C) The yield of interest depends on three sources of heterogeneity
(D) The yield does not depend on any source of heterogeneity

92. Which of the following is the null hypothesis in the omnibus hypothesis related to treatment effects in a completely randomized design?

(A) Inequality of all treatment effects
(B) Equality of the mean effect with a treatment effect
(C) Inequality of the mean effect and treatment effects
(D) Equality of all treatment effects

93. Which of the following is the distribution of the likelihood ratio test statistic pertaining to an analysis of variance model?

(A) Standard normal distribution
(B) Standard chi-square distribution
(C) The *F* distribution
(D) Student's *t* distribution

94. In simple linear regression, if *c* denotes the product-moment correlation coefficient between the variables and *r* denotes the regression coefficient, which of the following is **NOT** true?

(A) *c* and *r* have the same sign
(B) Ratio of *c* to *r* is ratio of the two standard deviations
(C) *c* and *r* have opposite sign
(D) $c = r$ if the two standard deviations are equal

95. In a simple linear regression model of *Y* on *X*, which of the following is equal to the regression coefficient?

(A) Slope of the line
(B) Vertical intercept of the line
(C) Horizontal intercept of the line
(D) Angle which the line makes with the horizontal axis

96. Which of the following is the mode of convergence in the central limit theorem?

(A) Convergence in distribution
(B) Convergence in probability
(C) Convergence in first mean
(D) Convergence in second mean

97. For the central limit theorem to hold for a sequence of independent random variables with the same non-degenerate probability distribution, which of the following condition should be satisfied by the sequence?

(A) The random variables should be non-negative
(B) The random variables should have first moment finite
(C) The random variables should have infinite first moment
(D) The random variables should have finite second moment

98. Given a sequence of independent random variables $X_1, X_2, \ldots$ having Uniform probability distribution over the interval (0, 1), which of the following is **NOT** true?

(A) $1 - X_1$ has Uniform probability distribution over the interval (0,1)
(B) Central limit theorem holds for the sequence
(C) $1 - X_2$ has Uniform probability distribution over the interval (– 1, 0)
(D) The random variables in the sequence have finite second moments

99. Given that X and Y are independent standard exponential random variables, which of the following is true?

(A) $X + Y$ has Exponential distribution with mean 2
(B) $X + Y$ has Uniform distribution over the interval (0, 1)
(C) $X + Y$ has Gamma distribution
(D) $X + Y$ has normal distribution

100. For a dataset, given that the sum of the first quartile and the third quartile is 20 and their difference is 4, which of the following is **NOT** true?

(A) The inter-quartile range is 4
(B) The first quartile is 8 and the third quartile is 12
(C) The first quartile is 12 and the third quartile is 16
(D) The difference between the first and third quartiles is 4

101. Given that variance of a random variable is zero, which of the following is true?

(A) The random variable takes only one value
(B) The random variable takes values -1 and 1 with equal probability
(C) The random variable takes at least two values of the form c and $-c$ with equal probabilities, where c is any positive number
(D) The random variable is a continuous random variable taking values in a non-empty interval

102. Given that probability of event A is less than probability of an event B, which of the following is **NOT** true?

(A) Probability of union of events A and B is greater than or equal to probability of A
(B) Probability of union of events A and B is greater than or equal to probability of B
(C) Probability of union of events A and B is greater than or equal to probability of intersection of events A and B
(D) Probability of union of events A and B is less than or equal to probability of intersection of events A and B

103. Given that X and Y are independent random variables with $E(X) = E(Y) = 0$, standard deviation of X equal to twice the standard deviation of Y, and standard deviation of Y equal to 2, which of the following is **NOT** true?

(A) Variance of $(X - Y)$ is equal to 20
(B) Variance of $(X + Y)$ is equal to 20
(C) Variance of $(X - Y)$ is equal to 12
(D) Variance of $(X + 2Y)$ is equal to 32

104. Given that probability of event B is not zero and A and B are independent events, which of the following is **NOT** true?

(A) Probability of A is equal to probability of A conditioned on B
(B) Probability of B is equal to probability of B conditioned on A
(C) Probability of A intersection B is equal to product of probability of A and probability of B
(D) Probability of A union B is equal to sum of probability of A and probability of B

105. When was the last Census held in India?

(A) 2010
(B) 2011
(C) 2022
(D) 2020

106. With reference to Simple Random Sampling (SRS), which of the following is **NOT** true?

(A) Probability of selecting a population unit in a simple random sample is the same for all the population units
(B) Probability of selecting a population unit in a simple random sample is not the same for all the population units
(C) SRS is an equal probability sampling
(D) All simple random samples are equal probability samples but not all equal probability samples are samples from simple random sampling

107. Which of the following is ISI, the premier institute established by Professor P.C.Mahalanobis?

(A) Indian Science Institute
(B) Indian Statistical Institute
(C) International Statistical Institute
(D) Indian Statistics Institution

108. Which of the following does **NOT** refer to a type of systematic sample?

(A) Linear systematic sample

(B) Circular systematic sample
(C) Systematic random sample
(D) Proportional systematic sample

109. Which of the following is **NOT** a property of an estimator?

(A) Unbiasedness
(B) Sufficiency
(C) Maximizing the likelihood
(D) Parsimony

110. Given a random sample from standard Bernoulli distribution with probability of success equal to p and probability of failure equal to $(1 - p)$, $0 < p < 1$, which of the following is **NOT** true?

(A) Sample mean is an unbiased estimator of p
(B) Sample mean is a consistent estimator of p
(C) There are estimators of p other than the sample mean
(D) Sample mean is the only consistent estimator of p

111. Given a random sample from Poisson distribution with parameter lambda, which of the following is **NOT** true?

(A) Sample mean is unbiased estimator of lambda
(B) Sample variance is unbiased estimator of lambda
(C) Half of sum of sample mean and sample variance is unbiased estimator of lambda
(D) One third of sum of sample mean and sample variance is unbiased estimator of lambda

112. Which of the following is true with reference to a likelihood function?

(A) It is a function of the data given the parameters
(B) It is a function of the parameters given the data
(C) It is neither a function of parameters nor a function of data
(D) It is function of the data and the parameters

113. Which of the following depicts income distribution in a population?

(A) Lorenz curve

(B) Fisher curve
(C) Rao curve
(D) Neyman curve

114. Which of the following is a statistical measure of income inequality in a population?

(A) Pearson coefficient
(B) Goldman coefficient
(C) Anderson coefficient
(D) Gini coefficient

115. Which of the following is termed as the ideal price index?

(A) Sen's price index
(B) Wilks' price index
(C) Fisher's price index
(D) Rao's price index

116. Which of the following is used to represent statistical data graphically?

(A) Root-and-Stem diagram
(B) Root-and-Flower diagram
(C) Stem-and-Flower diagram
(D) Stem-and-Leaf diagram

117. Which of the following is a method of estimation in Statistics?

(A) Least squares
(B) Least means
(C) Least coefficient of variation
(D) Least kurtosis

118. Which of these relate to confidence interval estimation?

(A) Confidence intervals are intervals involving a probability
(B) Confidence intervals are deterministic intervals not involving probability
(C) Confidence intervals are unbounded intervals always
(D) Confidence intervals cannot contain negative numbers

119. Which of the following is **NOT** a measure of spread of data?

(A) Maximum value minus minimum value
(B) Difference between the third quartile and the first quartile
(C) Difference between mean and median
(D) Mean of sum total of squared deviations of observations from the mean

120. Which of the following is the ministry responsible for Statistics in the Central Government of India?

(A) Ministry of Statistics and Programme Implementation
(B) Ministry of Government Statistics and Programme Implementation
(C) Ministry of Statistics and Programme Identification
(D) Ministry of Niti Aayog

121. Which of the following is **NOT** true for the infinite sum $1 - 1 + 1 - 1 + 1 - 1 + \ldots$?

(A) The infinite sum does not converge
(B) The infinite sum converges to 0
(C) The limit of the partial sum sequence of the infinite sum does not exist
(D) The partial sum sequence is oscillating

122. Which of the following is **NOT** a vector space?

(A) The set of all real numbers
(B) The set of all possible pairs of real numbers
(C) The set of all unit vectors in two dimensions
(D) The set containing only the null vector of a vector space

123. Which of the following is true with reference to the rank of a matrix?

(A) Rank of a matrix is equal to the number of non-zero rows in the matrix
(B) Rank of a matrix is equal to the number of linearly independent rows in the matrix
(C) Rank of a matrix is equal to the number of linearly dependent rows in the matrix
(D) Rank of a matrix is equal to the number of null rows in the matrix

124. Given that $F(x)$, x in the set of all real numbers, is the probability distribution function of a random variable X, which of the following is **NOT** true?

(A) F is defined on the set of all real numbers and takes values in the closed interval [0,1]
(B) F times F is a probability distribution function
(C) F can be a step function or a continuous function
(D) $F\left(\frac{1}{2}\right) = 0$

125. Given that $F(x)$, x in the set of all real numbers, is the probability distribution function of independent random variables X and Y, which of the following is **NOT** true?

(A) F times F is the probability distribution function of maximum of X and Y
(B) $(1 - F)$ times $(1 - F)$ is the probability distribution function of minimum of X and Y
(C) One minus ($(1 - F)$ times $(1 - F)$) is the probability distribution function of minimum of X and Y
(D) $(1 - F)$ is not a probability distribution function

126. $e^{\frac{i\pi}{2}}$ is equal to

(A) 0
(B) 1
(C) i
(D) $\frac{1}{2}$

127. If $X \sim U(0,2)$ and $Y = [X]$, the integer part of X, then $E(Y)$ is

(A) 1

(B) $\frac{1}{2}$

(C) 1.5

(D) 0

128. $\lim_{n\to\infty} e^{-i\pi n}$ is

(A) 1

(B) $-\infty$

(C) 0

(D) not converging

129. If $Z = \frac{1+i}{1-i}$, where $i = \sqrt{-1}$, then $|Z|$ is

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

130. The distribution function $F(x)$ of the random variable $X \sim U(0,1)$ is

(A) continuous and differentiable everywhere

(B) differentiable everywhere

(C) differentiable but not continuous everywhere

(D) continuous but not differentiable everywhere

131. If $X \sim N(0,1)$ and $Y = X^4$, then correlation between X and Y is

(A) $\frac{1}{4}$

(B) 1

(C) 0

(D) $\frac{1}{2}$

132. If 1, 3 are the eigen values of a square matrix A of order 2, then eigen values of A^4 are

(A) 1 and 4
(B) 1 and 12
(C) 1 and 81
(D) 1 and 6

133. If moment generating function of the random variable X is given by $M(t) = \frac{1}{3} + \frac{1}{6}e^{-3t} + \frac{1}{2}e^{4t}$, the n $p(X \geq 0)$ is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{5}{6}$

(D) $\frac{1}{2}$

134. Type-I beta distribution $B(m,n)$ is symmetric about $\frac{1}{2}$ if

(A) $m = n$
(B) $m > n$
(C) $m < n$
(D) $m = 2$ and $n = 3$

135. If X_1 and X_2 are independent $N(0,1)$ variates, then $P(X_1 > 2X_2)$ is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 1

(D) $\frac{1}{3}$

136. The minimum value of $f(x)=|x-1|+|x-2|+|x-7|,\ x\in R$ is

(A) 1
(B) 6
(C) 2
(D) 0

137. If $-3, -7, 4, 2$ is a random sample from $u(-\theta,\theta)$, $\theta>0$ then MLE of θ is

(A) 4
(B) 2
(C) 7
(D) 1

138. If X follows Standard Cauchy distribution, then $P(X>1)$ is

(A) $\frac{1}{2}$
(B) 1
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$

139. If $X_1, X_2, ...X_{1000}$ are independently and identically distributed random variables with $E(X_1)=1, V(X_1)=2,$ the $P\left(\frac{1}{1000}\sum_{j=1}^{1000} X_j \geq 1\right)$ is approximately equal to

(A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{1000}$
(D) $\frac{1}{100}$

140. Which of the following statement is **TRUE**?

(A) The only distribution for which mean = variance is the poisson distribution

(B) If mean of a distribution is 10 and variance =0 then $P(X=10)=1$

(C) The only mode of $U(0,1)$ distribution is $\frac{1}{2}$

(D) For *t*-distribution with 1 degrees of freedom, expectation is zero

141. The harmonic mean of the number 1, 2, 3 is

(A) 2

(B) $\frac{18}{11}$

(C) $\frac{13}{18}$

(D) $\frac{11}{18}$

142. The geometric mean of the observations 2, 4, 1, 8 is

(A) 2
(B) 1
(C) 6
(D) $2\sqrt{2}$

143. The weighted mean of *n* natural numbers with weights are the corresponding numbers is

(A) $\frac{n+1}{2}$

(B) $\frac{2n+1}{2}$

(C) $\frac{2n+1}{3}$

(D) $\frac{n(n+1)}{6}$

144. The appropriate measure of central tendency for the shoe size of 100 males is

(A) Mean
(B) median
(C) mode
(D) geometric mean

145. Variance of the numbers 1000, 1001, 1002 is

(A) 1000
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$

146. If X is a random variable with $P(X=-1)=P(X=1)=\frac{1}{2}$, then $V(X^4)$ is

(A) 4
(B) 2
(C) $\frac{1}{2}$
(D) 0

147. Let X be a Bernoulli random variable taking values 0 and 1. If $E(X)=3\,\mathrm{var}(X)$ then the probability that $X=1$ is

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{2}{5}$
(D) $\frac{3}{5}$

148. Let $\left(X_1, X_2, ..., X_n\right)$ be a random sample from a Poisson distribution with parameter λ and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the maximum likelihood estimate of $e^{-\lambda}$ is

(A) $\bar{X}$

(B) $\sum_{i=1}^{n} X_i$

(C) $e^{-\bar{X}}$

(D) $e^{-\sum_{i=1}^{n} X_i}$

149. If X follows an F-distribution with 5 and 10 degrees of freedom, then $\frac{1}{X}$ follows

(A) a chi-square distribution with 15 degrees of freedom

(B) an F-distribution with $\frac{1}{5}$ and $\frac{1}{10}$ degrees of freedom

(C) a student-t distribution with 15 degrees of freedom

(D) an F-distribution with 10 and 5 degrees of freedom

150. Given the joint probability density function of X and Y as

$$f(x, y) = \begin{cases} 4xy; & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases},$$

$P\left[0 < X < 0.5; 0.5 \le Y \le 1\right]$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{5}{16}$

(C) $\frac{3}{8}$

(D) $\frac{3}{16}$

ANSWER KEY									
Subject Name:		**STATISTICS**							
SI No.	**Key**	**SI No.**	**Key**	**SI No.**	**Key**	**SI No.**	**Key**	**SI No.**	**Key**
1	C	31	D	61	C	91	A	121	B
2	C	32	C	62	D	92	D	122	C
3	A	33	B	63	A	93	C	123	B
4	D	34	D	64	D	94	C	124	D
5	D	35	C	65	C	95	A	125	B
6	A	36	B	66	C	96	A	126	C
7	B	37	B	67	C	97	D	127	B
8	C	38	D	68	D	98	C	128	D
9	C	39	A	69	C	99	C	129	B
10	C	40	C	70	C	100	C	130	D
11	D	41	A	71	C	101	A	131	C
12	C	42	B	72	B	102	D	132	C
13	D	43	C	73	A	103	C	133	C
14	D	44	C	74	A	104	D	134	A
15	C	45	D	75	C	105	B	135	A
16	A	46	A	76	A	106	B	136	B
17	C	47	A	77	B	107	B	137	C
18	B	48	A	78	B	108	D	138	C
19	D	49	D	79	C	109	D	139	B
20	D	50	B	80	D	110	D	140	B
21	B	51	B	81	C	111	D	141	B
22	D	52	D	82	A	112	B	142	D
23	B	53	A	83	A	113	A	143	C
24	C	54	A	84	B	114	D	144	C
25	A	55	D	85	A	115	C	145	B
26	B	56	B	86	D	116	D	146	D
27	C	57	D	87	B	117	A	147	B
28	D	58	D	88	A	118	A	148	C
29	B	59	A	89	B	119	C	149	D
30	A	60	B	90	C	120	A	150	D