MATHEMATICS - PG LEVEL

1. The function $f(z) = \overline{z} + 2iz$ is

(A) nowhere analytic
(B) analytic everywhere
(C) analytic except at the origin
(D) analytic only at origin

2. Let n denote the number of solutions of the equation $z^2 + 2\overline{z} = 0$, where z is a complex number. Then, the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(A) 1
(B) $\frac{3}{2}$
(C) $\frac{4}{3}$
(D) 2

3. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}\left(p+iq\right)$, then p and q are roots of the equation

(A) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$
(B) $x^2 + \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$
(C) $x^2 + \left(\sqrt{3}-1\right)x + \sqrt{3} = 0$
(D) $x^2 - \left(\sqrt{3}-1\right)x + \sqrt{3} = 0$

4. If R is the radius of convergence of the series $\sum_{n=1}^{\infty} a_n z^n$, then the series $\sum_{n=1}^{\infty} n a_n z^{n-1}$ has the radius of convergence

(A) $\frac{1}{R}$
(B) $R-1$
(C) R
(D) 0

5. The area of the polygon, whose vertices are the non-real roots of the equation $\overline{z} = iz^2$, is

(A) $\frac{3\sqrt{3}}{4}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$

(D) $\frac{3}{4}$

6. If z is a complex number such that $|z| \geq 2,$ then the minimum value of $\left|z+\frac{1}{2}\right|$

(A) is equal to $\frac{5}{2}$

(B) lies in the interval $(1, 2)$

(C) is strictly greater than $\frac{5}{2}$

(D) is strictly greater than $\frac{3}{2}$ but less than 2

7. The continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ is

(A) 1

(B) 0

(C) 4π

(D) $\frac{\pi}{6}$

8. The value of $(-i^{i})$ is

(A) i

(B) 0

(C) $e^{\frac{\pi}{2}}$

(D) $ie^{\frac{\pi}{2}}$

9. If z is a complex number such that $|z| \leq 1,$ then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{1}{2}$

(D) $\frac{7}{2}$

10. The value of the integral $\int\limits_{|1-z|=1} \frac{e^z}{z^2-1}dz$ is

(A) 0
(B) πie
(C) $\pi e + \pi ie^{-1}$
(D) $e + e^{-1}$

11. The value of $\int_C \frac{dz}{z^2+6}$ where C is the boundary of $|z-i|=1$ is

(A) $2\pi i$
(B) $4\pi i$
(C) 0
(D) πi

12. Which of the following function $f(z)$ of the complex variable z is NOT analytic at all the points of the complex plane?

(A) $f(z) = z^2$
(B) $f(z) = \log z$
(C) $f(z) = e^z$
(D) $f(z) = \sin z$

13. The value of $\int_{|z|=2} \frac{dz}{z+3}$ is

(A) 0

(B) $-\frac{\pi}{2}$

(C) $2\pi i$

(D) $\frac{\pi}{2}$

14. The radius of convergence R of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is equal to

(A) 1
(B) ∞
(C) e
(D) $2e$

15. The value of $\int_C |z|\,dz,$ where C is the left half of the unit circle $|z|=1$ from $z=-i$ to $z=i$ is

(A) 0
(B) i
(C) $2i$
(D) $2\pi i$

16. The value of $\lim_{x\to 0}\left(\frac{1}{x}+\cot x\right)$ equals

(A) -1
(B) 0
(C) 1
(D) ∞

17. Let $P(x)$ be the particular solution of the differential equation $y'' + y = x\sin x$, then the value of $P\left(\frac{\pi}{2}\right)$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) 2π

(D) $\frac{\pi}{8}$

18. Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt} + py = re^{-qt}$, where $p > 0$, $q > 0$ and $r > 0$. Then, $\lim_{t\to\infty} y(t)$

(A) is 0
(B) is 1
(C) is -1
(D) does not exist

19. The number of positive roots of the equation $10x^3 - 7x - 4 = 0$ is

(A) 1
(B) 2
(C) 0
(D) 3

20. A person goes to the office either by car, bike, bus or train, probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}, \frac{1}{7}$ respectively. Probabilities that he reaches office late, if he takes car, bike, bus or train are $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reaches the office on time, then the probability he travelled by car is

(A) $\frac{1}{7}$

(B) $\frac{2}{7}$

(C) $\frac{2}{9}$

(D) $\frac{4}{7}$

21. The area of the curve $f(x) = 2\sqrt{x}$, bounded by $x = 1,\ x = 4$ and the X-axis in first quadrant is equal to the area of the rectangle bounded by $x = 1,\ x = 4$ and X-axis in first quadrant. Then the height of the rectangle is

(A) 3 units

(B) $\frac{28}{3}$ units

(C) $\frac{28}{6}$ units

(D) $\frac{28}{9}$ units

22. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\sin^2\left(\frac{\theta}{2}\right)$ is

(A) $\frac{\left|\vec{a}+\vec{b}\right|^2}{4}$

(B) $\frac{\left|\vec{a}-\vec{b}\right|^2}{4}$

(C) $\frac{\left|\vec{a}+\vec{b}\right|^2}{2}$

(D) $\frac{\left|\vec{a}-\vec{b}\right|^2}{2}$

23. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is

(A) 792
(B) 1594
(C) 924
(D) 1024

24. The solution of the differential equation $\frac{dy}{dx} = (1+x)\left(1+y^2\right)$ is

(A) $y = \tan\left(x^2 + x + c\right)$

(B) $y = \tan\left(2x^2 + x + c\right)$

(C) $y = \tan\left(x^2 - x + c\right)$

(D) $y = \tan\left(\frac{x^2}{2} + x + c\right)$

25. The value of $\lim_{n\to\infty}\left(\frac{n!}{(n+1)!-n!}\right)$ is

(A) ∞
(B) 1
(C) $\frac{1}{2}$
(D) 0

26. If $x \in [-1,1]$, then the value of $\left(2\sin^{-1}x + 2\cos^{-1}x\right)^2$ is

(A) π^2

(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi^2}{2}$

(D) 4

27. The minimum value of the function $f(x, y) = p^2x + q^2y$, where $xy = r^2$ is

(A) $2pqr$
(B) 1
(C) $p^2q^2r^2$
(D) $p^2q^2r^4$

28. For all complex numbers z on the curve $C_1 : |z| = 4,$ let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Thenthe curves

(A) C_1 and C_2 intersect at 4 points
(B) C_1 lies inside C_2
(C) C_1 and C_2 intersect at 2 points
(D) C_2 lies inside C_1

29. Let $A = \left[a_{ij}\right]_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $4a^2 + 7b^2$ is equal to

(A) 4
(B) 3
(C) 7
(D) 14

30. If the centroid of tetrahedron $OPQR$, where P, Q, R are given by (a, 1, 2), (2, b, 3) and (3, 1, c) respectively be(1, 2, −1), then the distance of $A(a, b, c)$ from origin is equal to

(A) $\sqrt{107}$
(B) $\sqrt{114}$
(C) $\sqrt{109}$
(D) $\sqrt{118}$

31. The product $(64)(64)^{\frac{1}{3}}(64)^{\frac{1}{9}}......\infty$ is equal to

(A) 8^3
(B) 4^3
(C) 8
(D) 0

32. The number of solutions of the equation $\sin(e^x) = 2^x + 2^{-x}$ is

(A) 2
(B) 0
(C) 1
(D) infinitely many

33. A Statistics book contains 200 pages. A page is selected at random. What is the probability that the number on the page selected is a perfect square?

(A) $\frac{7}{50}$

(B) $\frac{7}{100}$

(C) $\frac{7}{200}$

(D) $\frac{7}{25}$

34. The solution of the system of equations

$$5x + y + z = 7,$$
$$x + 5y + z = 7,$$
$$x + y + 5z = 7$$

is

(A) 1, 1, 1
(B) −1, −1, −1
(C) 1, −1, 1
(D) −1, 1, −1

35. When 51^{25} is divided by 13, the remainder obtained is

(A) 1
(B) 4
(C) 12
(D) 9

36. If the points with position vectors

$$\alpha\hat{i} + 10\hat{j} + 13\hat{k},$$
$$6\hat{i} + 11\hat{j} + 11\hat{k},$$
$$\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$$

are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(A) 49
(B) 36
(C) 25
(D) 16

37. If roots of the equation $x^2 + x + 1 = 0$ are α and β, then the equation whose roots are α^7 and β^4 is

(A) $x^2 - x - 1 = 0$
(B) $x^2 - x + 1 = 0$
(C) $x^2 + x - 1 = 0$
(D) $x^2 + x + 1 = 0$

38. The curvature of the unit circle $x^2 + y^2 = 9$ is

(A) 3
(B) $\frac{3}{2}$
(C) 0
(D) $\frac{3}{4}$

39. If $f(x) = \log_e\left(x + \sqrt{1 + x^2}\right)$, then $f^{-1}(x)$ is equal to

(A) $\frac{e^x - e^{-x}}{2}$
(B) $\frac{e^x + e^{-x}}{2}$
(C) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$
(D) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

40. If the term independent of x in the expansion of $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then a^3 is equal to

(A) 4
(B) 8
(C) 6
(D) 9

41. The function $f(z)=|z|^2+i\bar{z}$ is differentiable at

(A) i
(B) -1
(C) $-i$
(D) 0

42. Let (X_1, d_1) and (X_2, d_2) be metric spaces and let $f: X_1 \to X_2$ be a homeomorphism. Consider the following statements:

Statement I: If X_1 is compact, then X_2 is compact.

Statement II: If X_1 is complete, then X_2 is complete.

Then which of the following is true?

(A) Only Statement I
(B) Only Statement II
(C) Neither Statement I nor Statement II
(D) Both Statement I and Statement II

43. Let G be a cyclic group such that G has an element of infinite order. Then the number of elements of finite order in G is

(A) 0
(B) 1
(C) 2
(D) infinite

44. $\lim\limits_{n\to\infty} \dfrac{5\cos n^2}{n^5} =$

(A) 5
(B) 0
(C) ∞
(D) 1

45. Which of the following principal ideals is a maximal ideal of $\mathbb{Z}$?

(A) $\langle 9 \rangle$
(B) $\langle 8 \rangle$
(C) $\langle 15 \rangle$
(D) $\langle 11 \rangle$

46. Consider the bilinear transformation $f(z)=\dfrac{3z-4}{z-1}$. Then f is

(A) parabolic

(B) hyperbolic
(C) elliptic
(D) not parabolic

47. The sum of residues of $f(z) = \tan z$ at its poles inside $|z| = 2$ is

(A) 0
(B) −2
(C) 2
(D) −1

48. Consider $X = [0, 1]$ with the discrete metric. Then which of the following is not true?

(A) X is disconnected
(B) X is complete
(C) X is compact
(D) X is closed

49. If $f(z) = x^3 - 3xy^2 + 2y + iv(x, y)$ is analytic, then

(A) $v(x, y) = 3x^2y - y^3 + 2x$

(B) $v(x, y) = 3x^2y - y^3 + 2y$

(C) $v(x, y) = y^3 - 3x^2y + 2y$

(D) $v(x, y) = x^3 - 3xy^2 + 2x$

50. A particular solution of $y''' + y'' - y + 1 = -e^{-x}$ is a constant multiple of

(A) xe^{-x}

(B) $-x^2e^x$

(C) $-xe^x$

(D) x^2e^x

51. Let $K = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{n}, ...\right\} \subset \mathbb{R}$. If I and L denote the set of interior points and the limit points of K respectively, then

(A) $I \cap L = \{0\}$

(B) $I = L$
(C) $I \cap L = \phi$
(D) $I \cup L = \{0\}$

52. The bilinear transformation which maps the points 0, 1, ∞ in the z-plane onto the points $-i$, ∞, 1 in w-plane is

(A) $\frac{z-1}{z+i}$

(B) $\frac{z+i}{z-1}$

(C) $\frac{z-i}{z+1}$

(D) $\frac{z+1}{z-i}$

53. Let $S = \left\{(a,b) \in \mathbb{R}^2 \mid ab \in \mathbb{Q}\right\}$. Then S is

(A) countable
(B) bounded
(C) finite
(D) uncountable

54. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 2025x^2$. Then the function is

(A) continuous and bounded
(B) uniformly continuous
(C) unbounded but not uniformly continuous
(D) not differentiable at zero

55. The differential equation whose linearly independent solutions are $\sin 2x$, $\cos 2x$ and e^{2x} is

(A) $\left(D^3 + D^2 - 4D\right) y = 0$

(B) $\left(D^3 - 2D^2 + 4D + 8\right) y = 0$

(C) $\left(D^3+2D^2-4D+8\right)y=0$

(D) $\left(D^3-2D^2+4D-8\right)y=0$

56. Let $f(z)=\dfrac{\cos z}{2z\left(z^2-27\right)}$. Then $\displaystyle\int_{|z|=2} f(z)dz=$

(A) 0

(B) $-\dfrac{1}{27}$

(C) $-\dfrac{\pi i}{27}$

(D) $-\dfrac{2\pi i}{27}$

57. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\left(x^2+1\right)^{2025}$ is

(A) onto but not one-one
(B) one-one but not onto
(C) both one-one and onto
(D) neither one-one nor onto

58. The sequence $\sqrt{5},\sqrt{5+\sqrt{5}},\sqrt{5+\sqrt{5+\sqrt{5}}},\ldots$ converges to

(A) $\dfrac{1+\sqrt{33}}{2}$

(B) $\dfrac{1-\sqrt{21}}{2}$

(C) $\dfrac{1+\sqrt{21}}{2}$

(D) 5

59. If G is a simple graph of order n which is also$n-2$ regular, then

(A) G always connected
(B) n is even

(C) n is odd
(D) G does not exist

60. Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ be the matrix over $\mathbb{Z}_7$. Then the inverse of A is

(A) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 5 \\ 0 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$

61. If G is a cyclic group of order 2025, then which of the following is not an order of a subgroup of G?

(A) 81
(B) 25
(C) 35
(D) 45

62. A solution of $\left(1+x^2\right)y'+2xy-4x^2=0$ satisfying the initial condition $y(0)=0$ is

(A) $\dfrac{4x^3}{3\left(1+x^2\right)}$

(B) $\dfrac{2x^2}{8\left(1+x^2\right)}$

(C) $4x^3\left(1+x^2\right)$

(D) $12x+4x^3$

63. Which of the following statement(s)is/are NOT true?

Statement I: Binomial Distribution is a continuous distribution.
Statement II: In a Poisson distribution, the mean and standard deviation are equal

(A) Only Statement I
(B) Only Statement II
(C) Neither Statement I nor Statement II
(D) Both Statement I and Statement II

64. Which of the following is the real part of an analytic function?

(A) $u(x, y) = x^2 - y^2 - 2xy - 2x + 3$

(B) $u(x, y) = x^3 + y^3$

(C) $u(x, y) = x^4 - y^4 - xy$

(D) $u(x, y) = x^2 + y^2 + 2xy$

65. Which of the following statements about sequence (a_n) are true?

Statement I: If $a_n^2 \to a^2$, then $a_n \to a$.

Statement II: If a_n is bounded, then it has a convergent subsequence.

Statement III: If $\lim\limits_{n\to\infty} \frac{(a_1 + a_2 + \ldots a_n)}{n} = l$, then $\lim\limits_{n\to\infty} = l$

(A) Both Statement I and Statement II
(B) Only Statement II
(C) Both Statement II and Statement III
(D) Statement I, Statement II and Statement III

66. If G is a connected planar graph with 20 vertices and 25 edges, then the number of bounded faces in any embedding of G on the plane is

(A) 7
(B) 6
(C) 5
(D) 3

67. The equation of plane through the intersection of $2x - y + 2z = 3$ and $x + 2y + z = 2$ and passing through (1, 2, 1) is

(A) $7x + 5y + 4z = 8$
(B) $13x - 6y + 13z = 22$
(C) $8x - 6y + 5z = -2$
(D) $9x - 2y + 9z = 14$

68. Let A be a $n \times n$ complex matrix. Then which of the following statement(s) is/are true?

Statement I: If λ is an eigen value of A, then zero is an eigen value of $(A - \lambda I)^n$.

Statement II: If all the eigen values of A are zero, then A is a zero matrix.

Statement III: If $A - 7I = I$, then 6 is an eigen value of A

(A) Only Statement I
(B) Both Statement II and Statement III
(C) Both Statement I and Statement III
(D) Only Statement II

69. The number of permutations σ in S_7 satisfying $\sigma(2) = 7$ and $\sigma(4) = 5$ is

(A) 24
(B) 120
(C) 110
(D) 720

70. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ is convergent to

(A) 0

(B) $\frac{2}{3}$

(C) 1

(D) 4

71. Which of the following is NOT a metric on $\mathbb{R}$?

(A) $d_1(x, y) = |x - y|$

(B) $d_2(x, y) = \frac{|x - y|}{1 + |x - y|}$

(C) $d_3(x, y) = |x^3 - y^3|$

(D) $d_4(x, y) = \min\{-x, y\}$

72. Let R be a ring with characteristics n where$n \geq 2$. If M is the ring of all 2×2 matrices over R, then the characteristic of M is

(A) n
(B) 2
(C) $2n$
(D) $4n$

73. Which of the following differential equations is exact?

(A) $(y^3 - 3xy)\, dx + (x^2 - xy)\, dy = 0$
(B) $(x^7y^2 + 3y)\, dx + (3x^8y - x)\, dy = 0$
(C) $(x^2 - 2y^2)\, dx + (y^4 - 4xy)\, dy = 0$
(D) $2xydx + (3x^2 - y^2)\, dy = 0$

74. The rank of the matrix $\begin{pmatrix} 1 & 0 & 2 & 3 & 2 \\ 2 & 2 & -1 & 3 & 1 \\ 1 & 5 & -2 & 4 & 3 \\ 4 & -1 & 4 & 7 & 3 \end{pmatrix}$ is

(A) 4
(B) 3
(C) 2
(D) 5

75. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined as $f(n) = \begin{cases} 0, & n \text{ is prime} \\ n, & \text{otherwise} \end{cases}$. If D denote the set of points at which f is discontinuous, then D is

(A) $\mathbb{Z}$
(B) ϕ
(C) set of all primes
(D) $2\mathbb{Z}$

76. If G is a connected graph with 10 vertices and 27 edges, then

(A) the maximum degree of G is 2
(B) G is non-planar
(C) G is 6-connected
(D) G is a tree

77. Let $P = \{-p : p \text{ is prime}\} \subset \mathbb{Z}$. For any $x \in P$, the number of divisors of x in $\mathbb{Z}$ is

(A) one
(B) two
(C) three
(D) four

78. Let $F = x^3y^2z$ be a scalar valued function. Then the directional derivative of F at (1, 2, 3) is maximum along the direction

(A) $4\vec{i} + 3\vec{j} + 2\vec{k}$
(B) $18\vec{i} + 9\vec{j} + 3\vec{k}$
(C) $9\vec{i} + 3\vec{j} + \vec{k}$
(D) $18\vec{i} + 6\vec{j} + 2\vec{k}$

79. If $A = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ is a matrix with eigenvalues $\sqrt{6}$, then a and b are respectively

(A) 2 and -1
(B) 2 and -2
(C) 2 and 1
(D) -2 and 1

80. The infinite series $\sum_{n=1}^{\infty} \frac{3^n}{2^{n+3}}$

(A) diverges to ∞
(B) converges to $\frac{1}{4}$
(C) converges to $\frac{1}{32}$
(D) converges to $\frac{1}{5}$

81. The number of generators of the cyclic group $\mathbb{Z}_{25}$ is

(A) 20
(B) 24
(C) 5

(D) 6

82. The Laplace transform of e^{4t} is

(A) $\dfrac{1}{s+2}$

(B) $\dfrac{1}{s-2}$

(C) $\dfrac{1}{s+4}$

(D) $\dfrac{1}{s-4}$

83. Let $f:\mathbb{R}\setminus\{0\}\to\mathbb{R}$ be a continuous function such that $f'(x)=0$ for all $x\in\mathbb{R}\setminus\{0\}$. Then the range of f has

(A) uncountable number of points
(B) countably infinite number of points
(C) at most two points
(D) at most one point

84. Let $f(z)=\dfrac{e^{\frac{1}{z}}}{\left(9+z^2\right)(z-i)}$. Then f has

(A) an essential singularity at zero
(B) removable singularity at i
(C) pole of order two at 3
(D) simple pole at $3i$

85. Let V be an n dimensional vector space over F and $T:V\to V$ be a linear transformation. Then which of the following condition implies T is one-to-one?

(A) $\ker(T)=\phi$
(B) T is onto
(C) nullity of T is one
(D) nullity of T is n

86. If $(3,3,3,3,3)$ is the degree sequence of a graph G, then

(A) G is connected
(B) G is regular

(C) G is Eulerian
(D) no such Gexists

87. Which of the following equivalents are true for a graph G?

Statement I: G is complete if and only if $diam(G) = 1$

Statement II: G is 2-colorable if and only if G is bipartite

Statement III: G is connected if and only if $\overline{G}$ is disconnected

(A) Only Statement II
(B) Both Statement I and Statement II
(C) Both Statement II and Statement III
(D) Both Statement I and Statement III

88. The ordinary differential equation $\dfrac{dy}{dx} = \dfrac{2y}{x}$ with the initial condition $y(0) = 0$, has

(A) infinitely many solution
(B) no solution
(C) more than one but only finitely many solution
(D) unique solution

89. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 4 & 7 & 2 & 5 & 6 \end{pmatrix}$ be a permutation on S_7. Then σ^{13} is

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 2 & 7 & 1 & 6 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 1 & 3 & 6 & 7 & 4 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 3 & 6 & 7 & 4 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 1 & 4 & 3 & 2 & 6 & 5 \end{pmatrix}$

90. Let $S = \left\{ (x, y) \in \mathbb{R}^2 : xy > 0 \right\}$. Then S is

(A) connected but not compact
(B) not connected but compact
(C) neither connected nor compact
(D) both connected and compact

91. Which of the following is NOT a vector space?

(A) $\mathbb{R}\times\mathbb{R}$ over $\mathbb{Q}$
(B) $\mathbb{C}$ over $\mathbb{Q}$
(C) $\mathbb{Q}[i]$ over $\mathbb{Q}$
(D) $\mathbb{Q}$ over $\mathbb{R}$

92. Let $W=\left\{\begin{pmatrix} a & 1 & 0\\ 0 & b & 1\\ 0 & 0 & c\end{pmatrix} : a,b,c\in\mathbb{R}\right\}$. Then choose the appropriate option.

(A) There exists a matrix $A\in W$ whose rank is 2
(B) W is a subspace of $M_3(\mathbb{R})$ with dimension 4
(C) W is a subspace of $M_3(\mathbb{R})$ with dimension 3
(D) W is a subspace

93. Let $V=\mathbb{R}^3$ be a real inner product space with the usual inner product. A basis for the subspace $U^{\perp}$, where $U=\langle(1,-3,4)\rangle$ is

(A) {(3, 1, 0), (12, 4, 0)}
(B) {(3, –1, 0), (12, –4, 0)}
(C) {(–3, 1, 0), (4, 0, 1)}
(D) {(3, 1, 0), (–4, 0, 1)}

94. Which of the following permutation is an even permutation?

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5\\ 2 & 3 & 5 & 4 & 1\end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5\\ 3 & 2 & 1 & 4 & 5\end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5\\ 2 & 3 & 4 & 5 & 1\end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5\\ 1 & 3 & 4 & 5 & 1\end{pmatrix}$

95. If $\vec{A}=yz\vec{i}+zy\vec{j}+xy\vec{k}$, then the line integral $\int_C \vec{A}.d\vec{r}$ from (0, 0, 0) to (2, 4, 8), along the curve C; $x=t,\ y=t^2, z=t^3$ is

(A) 0
(B) 64
(C) 32
(D) −1

96. If $S = \{a + ib : a, b \in \mathbb{Z},\ b \text{ is even}\}$, then S is

(A) a subring and ideal of $\mathbb{Z}[i]$
(B) a subring but not an ideal of $\mathbb{Z}[i]$
(C) an ideal of $\mathbb{Z}[i]$ but not a subring of $\mathbb{Z}[i]$
(D) neither a subring nor an ideal of $\mathbb{Z}[i]$

97. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$. Then

(A) T is a one-to-one linear transformation
(B) T is an onto linear transformation
(C) T is not a linear transformation
(D) T is linear but neither one-to-one nor onto

98. Let $f : (\mathbb{Z}, +) \to (\mathbb{Q}^*, .)$ be a group homomorphism such that $f(3) = \frac{1}{3}$. Then the value of $f(-9)$ is

(A) 1
(B) 27
(C) −1
(D) $\frac{1}{27}$

99. If the probability that a batsman will hit a six is 60% and if 10 balls are bowled to him, then the mean and variance are respectively

(A) 4, 2.6
(B) 0.6, 0.24
(C) 6, 2.4
(D) 0.4, 0.16

100. $\int\limits_{|z|=1} \frac{\cos z}{(z-\pi)^3} =$

(A) 0
(B) $2\pi i$
(C) πi
(D) $4\pi i$

101. Let V be an n dimensional vector space over F and $S \subset V$. If $\langle S \rangle$ denotes the span of S, then which of the following is true?

(A) Dimension of $\langle S \rangle > n$
(B) If dimension of $\langle S \rangle < n$, then S is linearly dependent
(C) If S is a subspace of V, then $\dim\langle S \rangle \geq |S|$
(D) If dimension of $\langle S \rangle = n$, then S is linearly independent

102. The number of ways in which 7 distinct objects can be placed in a circle is

(A) 720
(B) 360
(C) 380
(D) 840

103. Let V be a vector space of all $n \times n$ complex matrices over $\mathbb{C}$. Then which of the followings are NOT a subspace of V?

(I) Set of all orthogonal matrices
(II) Set of all symmetric matrices
(III) Set of all diagonal matrices
(IV) Set of all hermition matrices

(A) Only I
(B) I, III and IV
(C) Both I and IV
(D) Both I and II

104. If a, b, c are sides of a triangle, then $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ are also sides of a triangle is

(A) always true
(B) sometimes true
(C) true after some conditions
(D) never true

105. The product of three positive reals is 1 and their sum is greater than the sum of their reciprocals. Then exactly one of them is greater than

(A) 0
(B) 1
(C) -1
(D) -2

106. If the fourth roots of unity are z_1, z_2, z_3 and z_4, then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ is equal to

(A) 1
(B) -1
(C) 4
(D) 0

107. Let $|z - z_1|^2 + |z - z_2|^2 = k$ be a circle. If $z_1 = 2 + 3i$ and $z_2 = 4 + 3i$ are the extremities of a diameter, then $k =$

(A) $\frac{1}{4}$
(B) 4
(C) 2
(D) $\frac{3}{4}$

108. All the roots of the equation $a_1z^3 + a_2z^2 + a_3z + a_4 = 3$ where $|a_i| \leq 1$, $i = 1, 2, 3, 4$ lie outside a circle with centre at origin. Then the radius of the circle is

(A) 1
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{4}{3}$

109. The region of the argand diagram defined by $|z - 1| + |z + 1| \leq 4$ is

(A) boundary and interior of a circle
(B) boundary and interior of an ellipse
(C) interior of an ellipse
(D) interior of a circle

110. The equation $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$, $b \in R$ represents a circle, if

(A) $|a| = b$
(B) $|a| \neq b$
(C) $|a|^2 > b$
(D) $|a|^2 < b$

111. Let n be an integer which leaves remainder one when divided by 3. Then $\left(1+i\sqrt{3}\right)^n+\left(1-i\sqrt{3}\right)^n$ equals

(A) $-(-2)^n$

(B) 2^{n+1}
(C) -2^{n+1}
(D) -2^n

112. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If their third roots are k_1 and k_2, then (k_1, k_2) is

(A) $(5,7)$
(B) $(-5,-7)$
(C) $(-5,7)$
(D) $(5,-7)$

113. The difference between two roots of the equation $x^3 - 13x^2 + 15x + 189 = 0$ is 2. Then the roots of the equation are

(A) $-3, 5, 7$
(B) $-3, -7, -9$
(C) $3, -5, 7$
(D) $-3, 7, 9$

114. In an examination of 9 papers, a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is

(A) 255
(B) 256
(C) 193
(D) 319

115. The number of triangles whose vertices are at the vertices of an octagon but none of whose sides happen to come from the octagon, is

(A) 16
(B) 28
(C) 56
(D) 70

116. Let A be the set of 4-digit number a_1, a_2, a_3, a_4 where $a_i \in \{1, 2, \ldots, 9\}$, $1 \le i \le 4$. Then $n(A)$ is equal to

(A) 32
(B) 84
(C) 126
(D) 210

117. The value of $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots\right)^2$ is

(A) 2
(B) –2
(C) 1
(D) –1

118. Let R be a relation over the set of real numbers defined by mRn if $mn \ge 0$. Then R is

(A) symmetric and transitive only
(B) reflexive and symmetric only
(C) a partial order relation
(D) an equivalence relation

119. Consider the relation R over the set of integers defined by $(x, y) \in R$ if and only if $|x - y| \le 1$. Then R is

(A) reflexive and transitive
(B) reflexive and symmetric
(C) symmetric and transitive
(D) an equivalence relation

120. Let A and B be two independent events such that $P(A)=\frac{1}{5}$, $P(A\cup B)=\frac{7}{10}$. Then $P(\overline{B})$ is equal to

(A) $\frac{3}{8}$
(B) $\frac{2}{7}$
(C) $\frac{7}{9}$
(D) $\frac{5}{7}$

121. It has been found that if A and B play a game 12 times. A wins 6 times, B wins 4 times and they draw twice. A and B takes part in a series of 3 games. The probability that they will win alternatively, is

(A) $\frac{1}{72}$

(B) $\frac{5}{72}$

(C) $\frac{19}{27}$

(D) $\frac{5}{36}$

122. All the spades are taken out from a pack of cards. From these cards, cards are drawn one by one without replacement till the ace of spades comes. The probability that the ace comes in the 4^{th} draw is

(A) $\frac{3}{13}$

(B) $\frac{12}{13}$

(C) $\frac{4}{13}$

(D) $\frac{1}{13}$

123. 6 ordinary unbiased dice are rolled. The probability that at least half of them will show at least 3, is

(A) $41\times\frac{2^4}{3^6}$

(B) $\frac{2^4}{3^6}$

(C) $20\times\frac{2^4}{3^6}$

(D) $21\times\frac{2^4}{3^6}$

124. If X follows a binomial distribution with parameters $n = 8$ and $p = \frac{1}{2}$, then $P\left(\left|X - 4\right| \le 2\right)$ is equal to

(A) $\frac{117}{128}$

(B) $\frac{118}{128}$

(C) $\frac{119}{128}$

(D) $\frac{120}{128}$

125. For two events A and B, it is given that $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$. Then

(A) A and B are mutually exclusive events

(B) A and B are dependent events

(C) $\overline{P\left(\frac{A}{B}\right)} = \frac{3}{4}$

(D) $\overline{P\left(\frac{A}{B}\right)} = \frac{1}{4}$

126. A box contains 24 identical balls of which 12 are white and remaining black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw, is

(A) $\frac{5}{64}$

(B) $\frac{27}{32}$

(C) $\frac{5}{32}$

(D) $\frac{1}{2}$

127. If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

(A) 5
(B) 3
(C) 4
(D) 2

128. If $\log_{|\sin x|}|\cos x| + \log_{|\cos x|}|\sin x| = 2$, then $|\tan x|$ is equal to

(A) ∞

(B) $\frac{1}{2}$

(C) 1

(D) 2

129. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(A) $\frac{1}{2}$

(B) 1

(C) $-\frac{1}{2}$

(D) $\frac{1}{8}$

130. If $\sin x + \sin^2 x = 1$, then $\cos^6 x + \cos^{12} x + 3\cos^{10} x + 3\cos^8 x$ is equal to

(A) 0
(B) ∞
(C) $\cos^3 x \sin^3 x$
(D) 1

131. If a and b are positive numbers such that $a > b,$ then the minimum value of $a\sec\theta - b\tan\theta$, where $\left(0 < \theta < \dfrac{\pi}{2}\right)$ is

(A) $\dfrac{1}{\sqrt{a^2-b^2}}$

(B) $\dfrac{1}{\sqrt{a^2+b^2}}$

(C) $\dfrac{1}{a}$

(D) $\sqrt{a^2-b^2}$

132. The minimum value of $27^{\cos 2x} \times 81^{\sin 2x}$ is

(A) 1

(B) $\dfrac{1}{9}$

(C) $\dfrac{1}{81}$

(D) $\dfrac{1}{243}$

133. If $f(x) = x^n$, then $f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots + \dfrac{(-1)^n f^{(n)}(1)}{n!}$ is

(A) 2^n
(B) 0
(C) 2^{n-1}
(D) 1

134. Let $f(x)$ and $g(x)$ be two differentiable functions on [0, 2] such that $f''(x) - g''(x) = 0,\ f'(1) = 2g'(1) = 4,\ f(2) = 3g(2) = 9.$ Then $f\left(\dfrac{3}{2}\right) - g\left(\dfrac{3}{2}\right) =$

(A) 0
(B) 2
(C) 10
(D) 5

135. Let $f(x)$ be a continuous double differentiable function such that $g(x) = f'(x)$ and $f''(x) = -f(x)$. If $f(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and $f(5) = 5$, then $f(10)$ is

(A) 0
(B) 5
(C) 10
(D) 25

136. The vectors $(x, y, 0)$, $(1, 0, z)$, $(1, 1, 0)$ are linearly independent in R^3 if

(A) $x \neq y$ and $z \neq 0$
(B) $x = y$ and $z \neq 0$
(C) $x = y = z$
(D) $x = y$ and $z = 0$

137. Let $X = (x_{ij})$ be a matrix if order $s \times t$and$x_{ij}=1$ for all i, j. Then rank (A) is

(A) s
(B) $s - t$
(C) 1
(D) 0

138. The number of subspaces of R^3 over R is

(A) 3
(B) 6
(C) 8
(D) 9

139. Let f be a monotone function on the open interval (a, b). Then $f$is

(A) continuous except possibly at a countable number of points in (a, b)
(B) continuous at all rational points in (a, b)
(C) continuous at all irrational points in (a, b)
(D) continuous at all points in (a, b)

140. Let H and K be two subgroups of group G. Then,

(A) $(H \cap K)\, a = aH \cap Ka,\ a \in G$
(B) $(H \cap K)\, a = Ha \cap Ka,\ a \in G$
(C) $(H \cap K)\, a \neq Ha \cap Ka,\ a \in G$

(D) $(H \cap K)\, a = aH \cap aK,\ a \in G$

141. If $A = [[-5 \quad -8 \quad 0][3 \quad 5 \quad 0][1 \quad 2 \quad -1]]$ then A is

(A) idempotent
(B) nilpotent
(C) involutory
(D) periodic

142. What is the equation of the surface of the cone with vertex at the origin and whose axis is along the line $x = y = z$?

(A) $x^2 + y^2 = z^2$
(B) $x^2 + y^2 - z^2 = 0$
(C) $x^2 + y^2 + z^2 = 0$
(D) $x^2 + y^2 - z^2 = 1$

143. Which of the following is an assumption of Jacobi's methods?

(A) The coefficient matrix has zeroes on its main diagonal
(B) The coefficient matrix has no zeroes on its main diagonal
(C) The rate of convergence is quite slow compared with other methods
(D) Iteration involved in Jacobi's method converges

144. A force $f = (10i + 8j - 5k)$ acts on a point $p(2, 5, 6)$. What will be the moment of the force about the point $Q(3, 1, 4)$?

(A) $-36i + 15j - 48k$
(B) $18i + 19j - 37k$
(C) $32i - 31j - 4k$
(D) Zero

145. What is the smallest positive integer in the set $\{30x + 170y + 1500z \mid x, y, z \in \mathbb{Z}\}$?

(A) 12
(B) 14
(C) 10
(D) 8

146. Find the ratio in which the xy plane divides the line joining the points $A(7, 4, -2)$ and $B(8, -5, 3)$.

(A) 2 : 5
(B) 2 : 3

(C) 5 : 3
(D) 3 : 2

147. Which of the following statement is false?

(A) The eigenvectors of a diagonal matrix are orthogonal
(B) The determinant of a matrix is equal to the product of its eigenvalues
(C) The eigenvectors of a symmetric matrix are orthogonal
(D) The eigenvectors of a diagonal matrix form a basis for the vector space

148. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1), then the value of a is

(A) 1
(B) 0
(C) 6
(D) –6

149. The number of divisors of 2100 is

(A) 35
(B) 36
(C) 26
(D) 53

150. Let f be continuous for $x \geq 0$, $f'(x)$ exists for $x > 0$, $f(0) = 0$ and f' is monotonically increasing. Let g be defined as $g(x) = \dfrac{f(x)}{x}$ $(x > 0)$. Then g is

(A) constant
(B) monotonically decreasing
(C) monotonically increasing
(D) periodic

ANSWER KEY

Subject Name: **MATHEMATICS PG**

SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key
1	A	31	A	61	C	91	D	121	D
2	C	32	B	62	A	92	A	122	D
3	A	33	B	63	D	93	D	123	A
4	C	34	A	64	A	94	C	124	C
5	A	35	C	65	B	95	B	125	C
6	B	36	B	66	B	96	B	126	C
7	A	37	D	67	D	97	C	127	B
8	C	38	A	68	A	98	B	128	C
9	A	39	A	69	B	99	C	129	C
10	B	40	B	70	C	100	A	130	D
11	C	41	C	71	D	101	D	131	D
12	B	42	A	72	A	102	A	132	D
13	A	43	B	73	C	103	C	133	B
14	C	44	B	74	B	104	A	134	D
15	C	45	D	75	B	105	B	135	B
16	B	46	A	76	B	106	D	136	A
17	D	47	B	77	D	107	B	137	C
18	A	48	C	78	C	108	C	138	C
19	A	49	A	79	B	109	C	139	A
20	A	50	A	80	A	110	C	140	B
21	D	51	C	81	A	111	A	141	C
22	D	52	B	82	D	112	B	142	B
23	C	53	D	83	C	113	D	143	B
24	D	54	C	84	D	114	B	144	A
25	D	55	D	85	B	115	A	145	C
26	A	56	C	86	D	116	C	146	B
27	A	57	D	87	B	117	C	147	A
28	A	58	C	88	A	118	D	148	C
29	C	59	B	89	B	119	B	149	B
30	D	60	D	90	C	120	A	150	C