QUESTION PAPER FOR COMMON ADMISSION TEST 2024
612 MATHEMATICS

1. The set of all points at which the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x-[x]$ is continuous is

   (A) $\mathbb{Z}$

   (B) $\dfrac{\mathbb{Q}}{\mathbb{Z}}$

   (C) $\dfrac{\mathbb{R}}{\mathbb{Z}}$

   (D) $\dfrac{\mathbb{R}}{\mathbb{Q}}$

2. The LUB of all real numbers in (0, 1) whose decimal expansion contains only 0 and 1 is

   (A) 1

   (B) 0.11

   (C) 0.1

   (D) $\dfrac{1}{9}$

3. Statement 1: If the sequences (x_n) and $(x_n y_n)$ are bounded, then (y_n) is bounded.

   Statement 2: If (x_n) and (y_n) are convergent sequences, then the sequence $S_n = \min\{x_n, y_n\}$ is convergent.

   Consider the above statements and choose the correct alternatives.

   (A) Both the statements are true
   (B) Statement 1 is false, Statement 2 is true
   (C) Statement 1 is true, Statement 2 is false
   (D) Both statements are false

4. Which of the following is **not** true for any vector space V ?

   (A) Basis of V is unique
   (B) If the basis of V is finite, then V is finite dimensional
   (C) If W is the subspace of V and dim V = dim W, then $V = W$
   (D) If V has a basis of n elements, then any set of $n + 1$ is linearly independent

5. Let $S, T : V \to W$ be a linear transformation such that ker (T) = ker (S) and Image (T) = Image (S). Then

(A) $S = T$ which is not identity
(B) S may not be equal to T
(C) Both S and T equal to the identity map
(D) $V = W$ always

6. A fair six-faced die is rolled 12 times. The probability that each face turns up twice is equal to

(A) $\frac{12!}{6!6!6^{12}}$

(B) $\frac{2^{12}}{2^6 6^{12}}$

(C) $\frac{12!}{2^6 6^{12}}$

(D) $\frac{12!}{6^2 6^{12}}$

7. Let $D = \{z \,/\, |z - 1| < 1\}$ and let $f : D \to \mathbb{C}$ be analytic function such that $f(1) = 1$ and $|f(1)| \geq |f(z)|\ \forall z \in D$. Then $f\left(\frac{3}{2}\right) =$

(A) 1
(B) i
(C) $1 + i$
(D) $\frac{1}{2}$

8. Let f be an entire function, $|f'(z)| \leq 3$ and $f'(0) = 2$. Then $f'(1) =$

(A) 1
(B) 2
(C) 3
(D) 0

9. If $\int_{|z|\le 2} \frac{dz}{z-1} = 2\pi i$, then $\int_{|z|\le 2} \frac{dz}{(z-1)^{10}} =$

(A) $\frac{\pi}{2}$

(B) $20\pi i$

(C) π

(D) 0

10. Let $f : [0, 1] \to [0, 1]$ be a continuous function. Then

(A) f is always one-one
(B) f is always onto
(C) there exists $c \in [0, 1]$ such that $f(c) = c$
(D) f is not always uniformly continuous

11. The maximum possible order of a simple graph G with minimum degree 3 and size 15 is

(A) 7
(B) 8
(C) 9
(D) 10

12. Which of the following sequences is **not** graphical?

(A) (8, 1, 1, 1, 1, 1, 1, 1, 1)
(B) (3, 3, 3, 3, 3, 3)
(C) (7, 4, 3, 2, 2, 1, 1)
(D) (8, 3, 3, 3, 3, 3, 3, 3, 3)

13. The radius of convergence of power series $f(x) = \sum_{n=2}^{\infty} \log(n) x^n$ is

(A) 0
(B) 1
(C) 3
(D) ∞

14. Which of the following set of functions from $\mathbb{R}$ to $\mathbb{R}$ is a vector space over $\mathbb{R}$?

(A) $S_1 = \left\{ f : \lim_{x \to 3} f(x) = 0 \right\}$

(B) $S_2 = \left\{ f : \lim_{x \to 3} f(x) = 1 \right\}$

(C) $S_3 = \left\{ f : \lim_{x \to 3} f(x) = 3 \right\}$

(D) $S_4 = \left\{ f : \lim_{x \to 3} f(x) = 4 \right\}$

15. Which of the following statements is true?

(A) If a finite group *G* contains an element of even order, then order of *G* is even
(B) If a finite group *G* contains an element of even order, then order of *G* is odd
(C) If a finite group *G* contains an element of even order, then order of *G* is prime
(D) If a finite group *G* contains an element of even order, then order of *G* is a perfect square

16. The function $f: \mathbb{C} \to \mathbb{C}$ defined by $f(z) = e^{z} + e^{-z}$ has

(A) finitely many zeros
(B) no zeros
(C) only real zeros
(D) has infinitely many zeros

17. Let $f : R \to R$ be defined as $f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$.

Then which of the following is true?

(A) f is discontinuous for all x
(B) f is continuous for all x
(C) f is discontinuous at $x = k\pi$, where k is an integer
(D) f is continuous at $x = k\pi$, where k is an integer

18. Two cards are drawn from a well-shuffled ordinary deck of 52 cards. The probability that they are both aces if the first card is replaced, is

(A) $\frac{1}{221}$

(B) $\frac{1}{169}$

(C) $\frac{1}{122}$

(D) $\frac{1}{196}$

19. If p is a polynomial with $p(0) = -1$, $p'(x) > 0 \ \forall \ x$, then

(A) p has more than one real root
(B) p has exactly one positive root
(C) p has exactly one negative root
(D) p has no real root

20. Consider the functions $f, g : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n + 2$, $g(n) = n^2 - 5$. Then

(A) both f and g are not one-one
(B) f is one-one but not g
(C) g is one-one but not f
(D) both f and g are one-one

21. The number of distinct homomorphisms from $\mathbb{Z}_5$ to $\mathbb{Z}_7$ is

(A) 0
(B) 1
(C) 5
(D) 7

22. The order of the group $GL(2, \mathbb{Z}_2)$ is

(A) 3
(B) 6
(C) 9
(D) 12

23. The number of isomorphisms from $\mathbb{Z}_6$ to S_3 is

(A) 0
(B) 1
(C) 2
(D) 3

24. Let f: $\mathbb{R} \to \mathbb{R}$ be a continuous map and let $Z(f) = \{x \in \mathbb{R} \mid f(x) = 0\}$. Then $Z(f)$ is always

(A) compact
(B) connected
(C) open
(D) closed

25. $\sin\frac{1}{x}$ is uniformly continuous in the interval

(A) $(0, \infty)$
(B) $[0, \infty)$
(C) $[1, \infty)$
(D) $(-1, 1)$

26. The function $f(x) = \begin{cases} x^2 - 3x + 2, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{Q}^c \end{cases}$ is continuous at

(A) exactly one point
(B) exactly two points
(C) exactly three points
(D) all integers

27. $\lim_{x \to \infty} (\cos x)^{1/x^2} =$

(A) 0
(B) $\frac{e}{2}$
(C) $\frac{1}{\sqrt{e}}$
(D) e^2

28. $\lim_{x\to\infty}\sum_{k=1}^{n}\frac{n}{k^2+n^2} =$

(A) e
(B) $\frac{\pi}{4}$
(C) $\frac{2}{3}$
(D) ∞

29. The dimension of the vector space $\{(x, y, z, w) \in \mathbb{R}^4 : x + y = z + w\}$ is

(A) 1
(B) 2
(C) 3
(D) 4

30. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = \cos z$. Then

(A) $|f(z)| \leq 1$
(B) $|f(z)| \leq \pi$
(C) $|f(z)| \leq |z|$
(D) f is unbounded

31. Let F_n be finite set with n elements. Then the number of one-one maps from F_5 to F_7 is

(A) 35
(B) $\binom{7}{5}$
(C) 5!
(D) $5!\binom{7}{5}$

32. Let $\alpha_1, \alpha_2, \ldots \alpha_{2023}$ be the roots of the equation $1 + x^{2023} = 0$. Then the value of the product $(1+\alpha_1)(1+\alpha_2)\ldots(1+\alpha_{2023})$ is

(A) 0
(B) 1
(C) 2
(D) 2023

33. Let $D = \{z \in \mathbb{C} ; |z| < 1\}$ and $f : D \to \mathbb{C}$ be defined by

$$f(z) = z - 25z^3 + \frac{z^5}{5!} - \frac{z^7}{7!} + \frac{z^9}{9!} - \frac{z^{11}}{11!}.$$

Statement A: f has three zeros (counting multiplicity) in D.

Statement B: f has one zero in $U = \{z \in \mathbb{C} ; \frac{1}{2} < |z| < 1\}$.

Then

(A) Both Statement A and Statement B are true
(B) Statement A is true and Statement B is false
(C) Statement A is false and Statement B is true
(D) Both Statement A and Statement B are false

34. Let G be a finite group and let $a \in G$ be an non identity element such that $a^{20} = e$. Which of the following cannot be the possible order of G?

(A) 12
(B) 9
(C) 20
(D) 15

35. Let V be a 7 dimensional vector space and W and Z be subspaces of dimensions 4 and 5 respectively. Which of the following is not possible for $\dim(W \cap Z)$?

(A) 1
(B) 2
(C) 3
(D) 4

36. Let p be a polynomial of degree $2n + 1$, $n \geq 1$ with real coefficients. Then p has

(A) exactly $2n + 1$ fixed points
(B) at least one fixed points
(C) n fixed points
(D) at most one fixed points

37. The eigen values of $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are

(A) $\cos\theta$ and $\sin\theta$
(B) $e^{i\theta}$ and $e^{-i\theta}$
(C) 1 and 2
(D) $\tan\theta$ and $\cot\theta$

38. Let $f : [0, 1] \rightarrow [0, 1]$ be continuous and $f(0) = 0$ and $f(1) = 1$. Then f is necessarily

(A) injective but not surjective
(B) surjective but not injective
(C) bijective
(D) surjective

39. Let $P = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 0\}$ and $A : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation satisfying $A(v) = 0 \ \forall \ v \in P$ and also $A(0, 0, 1) = (0, 0, 0)$. Then

(A) dimension of null space of A is 2
(B) A is the zero linear transformation
(C) Image $A = \mathbb{R}^3$
(D) dimension of the image of A is 2

40. If $\lim_{x \to 0} \left(\frac{1+cx}{1-cx} \right)^{\frac{1}{x}} = 4$, then $\lim_{x \to 0} \left(\frac{1+3cx}{1-3cx} \right)^{\frac{1}{x}} =$

(A) 2
(B) 4
(C) 16
(D) 64

41. The equation $\left(x^2 + y^2 - 1\right)\frac{\partial^2 y}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \left(x^2 + y^2 - 1\right)\frac{\partial^2 u}{\partial y^2} = 0$ is

(A) parabolic in the region $x^2 + y^2 > 2$
(B) hyperbolic in the region $x^2 + y^2 > 2$
(C) elliptic in the region $0 < x^2 + y^2 < 2$
(D) hyperbolic in the region $0 < x^2 + y^2 < 2$

42. Which of the following number can be an order of a permutation σ of 11 elements such that σ does not fix any elements?

(A) 14
(B) 15
(C) 16
(D) 17

43. The last digit of $(24)^{2024}$ is

(A) 0
(B) 2
(C) 4
(D) 6

44. Let $f_n(x)=\dfrac{x^n}{1+x},\ g_n(x)=\dfrac{x^n}{1+nx}$ for $x\in[0,1]$. Then

(A) both $\{f_n\}$ and $\{g_n\}$ converge uniformly
(B) only $\{f_n\}$ converges uniformly
(C) only $\{g_n\}$ converges uniformly
(D) both $\{f_n\}$ and $\{g_n\}$ do not converge uniformly

45. Let $f:[0,1]\to\mathbb{R}$ be continuous function and $f(0)=0$ and $f(1)=1$. Then

I: there exists $c\in[0,1]$ such that $f'(c)=1$
II: there exist $c_1, c_2\in[0,1]$ such that $f'(c_1)+f'(c_2)=2$

Then

(A) only I is true
(B) only II is true
(C) both I and II are true
(D) both I and II are false

46. The number of vertices in polyhedron having 40 edges and 12 faces is

(A) 12
(B) 15
(C) 20
(D) 30

47. The chromatic number of a simple connected graph of order n which does not contain any odd length cycle is

(A) $n-1$
(B) 3
(C) 2
(D) n

48. Let ω be the 7^{th} root of unity. Then the cubic polynomial with integer coefficients having $\omega + \omega^{-1}$ as a root is

(A) $x^3 - 7 = 0$
(B) $x^3 + x^2 - 2x - 1 = 0$
(C) $x^3 + 2x^2 + 2x + 1 = 0$
(D) $x^3 + 7 = 0$

49. If the scalar product (dot product) of two unit vectors is zero, they are

(A) linearly dependent
(B) part of an orthonormal basis
(C) pointing in the same direction
(D) at an angle of 180 degrees to each other

50. The matrix $\begin{pmatrix} 1 & 1.00001 & 1 \\ 1.00001 & 1 & 1.00001 \\ 1 & 1.00001 & 1 \end{pmatrix}$ has

(A) all eigen values positive
(B) one positive eigen value and one negative eigen value
(C) all eigen values zero
(D) all eigen values negative

51. The values of i^i in the form $a + bi$

(A) $e^{\frac{-\pi}{2}}$

(B) $\left\{ e^{\frac{-k\pi}{2}} \,|\, k \in \mathbb{Z} \right\}$

(C) $\cos i + i \sin i$

(D) $\left\{ e^{\frac{-\pi}{2} + 2k\pi} \,|\, k \in \mathbb{Z} \right\}$

52. The equation $y\frac{\partial^2 u}{\partial x^2}+4x\frac{\partial^2 u}{\partial y^2}$ is hyperbolic in the quadrants

(A) I and II
(B) III and IV
(C) I and III
(D) II and IV

53. Let $f(z)$ be a non constant entire function. Which of the following is true?

(A) $\text{Re} f(z) = \text{Im} f(z)$
(B) $|f(z)| < 1$
(C) $\text{Im} f(z) < 0$
(D) $f(z) \neq 0$

54. If $A = \left(0, \frac{1}{10}\right)$ in the metric space $M = (0, 1)$ with the usual distance metric, then $\overline{A}$

(A) $\left(0, \frac{1}{10}\right)$

(B) $\left(0, \frac{1}{10}\right]$

(C) $\left[0, \frac{1}{10}\right]$

(D) $\left[0, \frac{1}{10}\right)$

55. The dimension of the vector space of all symmetric matrices of order $n \times n$ $(n \geq 2)$ with real entries and trace equals to zero is

(A) $\frac{n^2-n}{2}-1$

(B) $\frac{n^2+n}{2}-1$

(C) $\frac{n^2-2n}{2}-1$

(D) $\frac{n^2+2n}{2}-1$

56. The number of 4 digit numbers with no two digits common is

(A) 2536
(B) 3536
(C) 4536
(D) 5536

57. The set of all matrices with trace 5 is

(A) a vector space of dimension $n^2 - 1$
(B) a vector space of dimension $n^2 - 5$
(C) a vector space of dimension n
(D) not a vector space

58. The number of 8 digit numbers that can be formed using 1, 2, 3, 4 is

(A) 8!
(B) 4^8
(C) 8^4
(D) 4!

59. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0$ is

(A) $\{1, x, e^x, e^{-x}\}$
(B) $\{1, x, e^{-x}, xe^{-x}\}$
(C) $\{1, x, e^x, xe^x\}$
(D) $\{1, x, e^x, xe^{-x}\}$

60. Which of the following functions is uniformly continuous?

(A) $f(x) = \sin^2 x,\ x \in \mathbb{R}$
(B) $f(x) = \frac{1}{x},\ x \in (0,1)$
(C) $f(x) = x^2,\ x \in \mathbb{R}$
(D) $f(x) = x + \frac{1}{x},\ x \in \mathbb{R}$

61. The interior of the set $\left\{R \in \mathbb{Q} : 0 < r < \sqrt{2}\right\}$ is

(A) $\mathbb{Q}$
(B) $\mathbb{R}$
(C) ϕ
(D) $\mathbb{Q} - \{0\}$

62. The Laplace transform of the equation $\int_0^t \int_0^t \int_0^t (t \sin t) dt \; dt \; dt$ is

(A) $\dfrac{2}{s^2(s^2+1)^2}$

(B) $\dfrac{2}{s^2(s+1)}$

(C) $\dfrac{2}{s(s+1)^2}$

(D) $\dfrac{2}{s^2(s^2+1)}$

63. If $G \neq \{e\}$ is a group having no proper subgroup, then G is a

(A) cyclic group of prime order
(B) cyclic group of even order
(C) cyclic group of odd order
(D) abelian group of even order

64. In a group of 100 people, each one knows at least 67 other people. Then the minimum number of people who are mutually friends is

(A) 0
(B) 2
(C) 3
(D) 4

65. Let A be the following subset of

$$\mathbb{R}^2;\ A = \left\{(x, y) : (x+1)^2 + y^2 \le 1\right\} \cup \left\{(x, y) : y = x \sin\frac{1}{x}, x > 0\right\}.$$

Then

(A) A is compact
(B) A is connected
(C) A is bounded
(D) A is not connected

66. Consider the three statements:

(I) $n^2 + n$ is divisible by 2.
(II) $n^3 - n$ is divisible by 3.
(III) $n^5 - 5n^3 + 4n$ is divisible by 5.

Which of the following is true?

(A) Only (I)
(B) (I) and (II)
(C) (I) and (III)
(D) (I), (II) and (III)

67. The order of the permutation (1 4 7) (2 5) in the symmetric group S_{12} is

(A) 7
(B) 5
(C) 6
(D) 12

68. The order of the coset $\overline{3}$ in the quotient group $\dfrac{\mathbb{Z}_{10}}{\langle 4 \rangle}$ is

(A) 2
(B) 3
(C) 4
(D) 5

69. Let G be a group of order 289. Then G is

(A) cyclic
(B) abelian
(C) not cyclic
(D) non-abelian

70. Let G be a group of order 40. Then H and K be two subgroups of G of orders 4 and 5. Then the order of the quotient group $G/H \cdot K$ is

(A) 1
(B) 2
(C) 10
(D) 8

71. Let G be the group of mappings $f_{x,y} : R \to R$ defined by $f_{x,y}(a) = xa + y$ for all $a \in R$. Then the group inverse of $f_{2,3}$ is

(A) $f_{\frac{1}{2}, \frac{-3}{2}}$

(B) $f_{\frac{1}{2}, \frac{3}{2}}$

(C) $f_{\frac{-1}{2}, \frac{3}{2}}$

(D) $f_{\frac{3}{2}, \frac{-1}{2}}$

72. The group $\mathbb{Z}_2 \times \mathbb{Z}_5$ is

(A) not cyclic
(B) cyclic
(C) abelian but not cyclic
(D) non-abelian

73. Every non-trivial subgroup of the group of integers with respect to addition is

(A) non-abelian
(B) finite
(C) non-cyclic
(D) infinite

74. Let G be a group of order 28 with an element of order 7. Then the number of elements of order 7 in G is

(A) 14
(B) 7
(C) 6
(D) 4

75. Let G be the symmetric group of degree 3. Then the number of subgroups of G is

(A) 2
(B) 4
(C) 5
(D) 6

76. Let $R = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \text{ are real numbers} \right\}$. Then R under matrix addition and matrix multiplication is a

(A) field
(B) non-commutative ring
(C) commutative ring but not a field
(D) not a ring

77. Let $X = \left(x_{ij}\right)$ be a matrix of order $m \times n$, where $x_{ij} = 1$ for all i, j. Then $rank(X)$ is

(A) $m + n$
(B) m
(C) n
(D) 1

78. The vectors (m, n, 0), (1, 0, p) and (1, 1, 0) are linearly independent in R^3 if

(A) $p \neq 0$ and $m \neq n$
(B) $p \neq 0$ and $m = n$
(C) $m \neq n$
(D) $m = n = p$

79. The number of non-trivial subspaces of $\mathbb{R}^3$ over $\mathbb{R}$ is

(A) 6
(B) 3
(C) 2
(D) ∞

80. If the point $A(3, 3)$ is shifted by a distance $\sqrt{2}$ unit parallel to the line $x = y$, then the coordinates of A in the new position is

(A) (5, 4)
(B) $\left(3+\sqrt{2}, 3+\sqrt{2}\right)$
(C) (3, 2)
(D) (2, 3)

81. The slopes of the lines represented by $x^2 + 5hxy + 2y^2 = 0$ are in the ratio 2:3, then h equals

(A) $\pm\frac{1}{\sqrt{2}}$
(B) $\pm\frac{1}{\sqrt{3}}$
(C) ± 2
(D) ± 3

82. Let I be the ideal generated by 4 in the ring of integers. Then I is

(A) a maximal ideal
(B) a prime ideal
(C) neither maximal nor prime
(D) prime but not maximal

83. Let R be a commutative ring of order 102 and let I be an ideal of order 34 in R. Then the quotient ring R/I is

(A) a commutative ring
(B) an integral domain
(C) a field
(D) a non-commutative ring

84. An example of a non-commutative ring is the ring of

(A) integers
(B) rationals
(C) quaternions
(D) modulo classes

85. Consider the ring $S = \left\{ \frac{a}{b} : a, b \in \mathbb{Z} \text{ with } b \text{ is odd} \right\}$ with respect to addition and multiplication of rationals. Then S has

(A) infinitely many maximal ideals
(B) finitely many maximal ideals
(C) no maximal ideal
(D) a unique maximal ideal $S = \left\{ \frac{a}{b} \in S : a, b \in \mathbb{Z} \text{ with } a \text{ is even} \right\}$

86. Consider the polynomial ring $R[x]$ where R is the field of real numbers. A maximal ideal in $R[x]$ is an ideal generated by

(A) an irreducible polynomial
(B) a reducible polynomial
(C) a constant polynomial
(D) a polynomial

87. If $5x - 12y - 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is

(A) 1
(B) 2
(C) 4
(D) 6

88. The vertex of the parabola $x^2 + 2y = 8x - 7$ is

(A) $\left(\frac{9}{2}, 0\right)$
(B) $\left(4, \frac{9}{2}\right)$
(C) $\left(2, \frac{9}{2}\right)$
(D) $\left(4, \frac{7}{2}\right)$

89. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

90. If the fourth roots of unity are z_1, z_2, z_3, z_4, then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ is equal to

(A) 1
(B) 0
(C) i
(D) $-i$

91. The equation $|z + 1 - i| = |z + i - 1|$ represents a

(A) pair of straight lines
(B) circle
(C) parabola
(D) hyperbola

92. The radius of the circle $\frac{|z-i|}{|z+i|} = 3$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{3}{11}$

(C) $\frac{3}{4}$

(D) 5

93. The equation $z\overline{z}+(1-3i)z+(1+3i)\overline{z}+6=0$ represents a circle of radius

(A) 2
(B) $\sqrt{2}$
(C) 3
(D) $\sqrt{3}$

94. Solution of the differential equation $x\,dy - y\,dx = 0$ represents a

(A) parabola whose vertex is the origin
(B) circle whose center is the origin
(C) rectangular hyperbola
(D) straight line passing through the origin

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

(A) $y = 2$
(B) $y = 2x$
(C) $y = 2x - 4$
(D) $y = 2x^2 - 4$

96. If $y = x^{(\log x)^{\log\log x}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y\log y}{x\log x}(2\log\log x+1)$

(B) $\frac{x\log x}{y\log y}(2\log\log x+1)$

(C) $\frac{2y\log y}{x\log x}(\log\log x+1)$

(D) $\frac{2x\log x}{y\log y}(\log\log x+1)$

97. If $f(x) = \sin\left(\frac{\pi}{2}[x] - x^3\right), 2 < x < 3$ and $[x]$ denotes the greatest integer less than or equal to x, then $f'\left(\sqrt[3]{\frac{\pi}{2}}\right)$ is equal to

(A) ∞
(B) -1
(C) 1
(D) 0

98. Let $f(x)$ be a function defined for all $x \in R$. If f is differentiable and $f\left(x^3\right) = x^5$ for all $x \in R (x \neq 0)$, then $f'(27)$ is equal to

(A) 0
(B) 5
(C) 15
(D) 25

99. If $f(x) = |x-1|$ and $g(x) = f\left[f\{f(x)\}\right]$ then, for $x > 2$, $g'(x)$ is equal to

(A) -1 if $2 < x < 3$
(B) 1 if $2 < x < 3$
(C) 1 for all $x > 2$
(D) 0

100. Let z be a function of x and y. If $x^x y^y z^z = 2$, then $\frac{\partial z}{\partial x}$ is equal to

(A) $\frac{1+\log x}{1+\log z}$
(B) $-\frac{1+\log x}{1+\log z}$
(C) $-\frac{1-\log x}{1+\log z}$
(D) $\frac{1+\log x}{1-\log z}$

101. If $f(x)=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$ then $f'(e)$ is equal to

(A) 0

(B) e

(C) $\frac{1}{e}$

(D) 1

102. Solution of the differential equation $\cos x\,dy = y(\sin x - y)dx,\ 0<x<\frac{\pi}{2}$, is

(A) $\sec x = \tan x + c$

(B) $y\sec x = \tan x + c$

(C) $\tan x = (\sec x + c)y$

(D) $y\tan x = \sec x + c$

103. The solution of the differential equation $x\frac{dy}{dx} = 2y + x^3e^x$, with $y(1)=0$, is

(A) $y = x^2(e^x - e)$

(B) $y = x^3(e - e^x)$

(C) $y = x^2(e - e^x)$

(D) $\tan x = (\sec x + c)y$

104. The integrating factor of the differential equation $(y\log y)dx = (\log y - x)dy$ is

(A) $\frac{1}{\log y}$

(B) $\log(\log y)$

(C) $\log y$

(D) $1 + \log y$

105. If $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1,$ then $|\vec{a} - \vec{b}|$ is equal to

(A) 1
(B) $\sqrt{3}$
(C) $\sqrt{2}$
(D) 2

106. The number of distinct real values of λ , for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k},\ \hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) 0
(B) 1
(C) 2
(D) 3

107. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3,\ |\vec{b}| = 4$ and $|\vec{c}| = \sqrt{37},$ then the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{6}$
(D) $\frac{\pi}{3}$

108. The sum of 20 terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + ...$ is

(A) $220\sqrt{2}$
(B) $210\sqrt{2}$
(C) $300\sqrt{2}$
(D) $320\sqrt{2}$

109. If $x^2 - 4$ is a factor of $x^4 + ax^3 + x - b$, then

(A) $a = 3,\ b = -1/4$
(B) $a = -1,\ b = 16$
(C) $a = 2/5,\ b = 16$
(D) $a = -1/4,\ b = 16$

110. The coefficient of x^6 in $\left\{(1+x)^6+(1+x)^7+...+(1+x)^{15}\right\}$ is

(A) $^{16}C_9$

(B) $^{16}C_5-^{6}C_5$

(C) $^{16}C_6-1$

(D) $^{16}C_6-^{6}C_5$

111. The domain of real valued function $f(x)=\sqrt{x-1}+\sqrt{5-x}$ is

(A) [1, 5]
(B) [−1, 5]
(C) [0, 5]
(D) [1, ∞]

112. Total number of solutions to the equation $2^{\cos x}=|\sin x|$ for $x\in[0,2\pi]$ is

(A) 0
(B) 1
(C) 2
(D) 4

113. Consider the function $g:N\to N$ defined by $g(x)=x-(-1)^x$ for all $x\in N$. Then g is

(A) one-to-one and onto
(B) one-to-one but not onto
(C) onto but not one-to one
(D) neither one-to-one nor onto

114. The value of $\lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is

(A) $\frac{1}{\sqrt{2\pi}}$

(B) $\frac{1}{\sqrt{\pi}}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

115. If $f(9) = 9$ and $f'(9) = 2$, then $\lim_{x \to 9} \dfrac{3 - \sqrt{f(x)}}{3 - \sqrt{x}}$ is

(A) 0
(B) 1
(C) −1
(D) 2

116. Value of $\lim_{x \to \frac{\pi}{2}} \dfrac{\left[\frac{x}{2}\right]}{\log(\sin x)}$ where $[x]$ is the greatest integer function, is

(A) does not exist
(B) equal to 1
(C) equal to −1
(D) equal to 0

117. The number of discontinuities of the greatest integer function $f(x) = [x]$ for $x \in \left[-\frac{7}{2}, 100\right]$ is

(A) 104
(B) 103
(C) 102
(D) 101

118. The function $f(x) = x - \left[x - x^2\right]$ is

(A) continuous at $x = 1$
(B) discontinuous at $x = 1$
(C) not defined at $x = 1$
(D) not defined at many points

119. The equation of the normal to the curve $y = (1+x)^y + \sin^{-1}\left(\sin^2 x\right)$ at $x = 0$ is

(A) $x + y = 2$
(B) $x + y = 1$
(C) $x - y = 1$
(D) $x - y = 2$

120. If the roots of the equation $x^3 - ax^2 + 4x - 8 = 0$ are real and positive, then the minimum value of a is

(A) 2
(B) $2\sqrt[3]{4}$
(C) $3\sqrt[3]{4}$
(D) 6

121. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. Then the largest value which $f(2)$ can attain is

(A) 7
(B) 5
(C) 3
(D) 2

122. If $\int f(x)dx = f(x)$, then $\int \left(f(x)\right)^2 dx$ is equal to

(A) $\frac{1}{2}\left(f(x)\right)^2$
(B) $\left(f(x)\right)^3$
(C) $\frac{\left(f(x)\right)^3}{3}$
(D) $\left(f(x)\right)^2$

123. The solution set of the equation $\begin{bmatrix} 2 & 3 & x \\ 2 & 1 & x^2 \\ 6 & 7 & 3 \end{bmatrix} = 0$ is

(A) ϕ
(B) {0, 1}
(C) {1,–1}
(D) {1,–3}

124. The probability that a man will live 10 more years is $\frac{3}{5}$ and the probability that his wife will live 10 more years is $\frac{2}{7}$. Then the probability that none of them will be alive after 10 years is

(A) $\frac{2}{5}$

(B) $\frac{2}{7}$

(C) $\frac{3}{5}$

(D) $\frac{5}{7}$

125. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is

(A) $\frac{1}{2}$

(B) $\frac{5}{9}$

(C) $\frac{4}{9}$

(D) $\frac{2}{3}$

126. Seven balls are drawn simultaneously from a bag containing 5 white and 6 green balls. The probability of drawing 3 white and 4 green balls is

(A) $\frac{7}{^{11}C_7}$

(B) $\frac{^5C_3\,^6C_2}{^{11}C_7}$

(C) $\frac{^5C_3 +^6 C_2}{^{11}C_7}$

(D) $\frac{^6C_3\,^5C_4}{^{11}C_7}$

127. In a triangle ABC, $a = 7$, $b = 9$, and $\sin A = \frac{7}{9}$, then B is equal to

(A) 60°
(B) 90°
(C) 45°
(D) 70°

128. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

(A) 0
(B) 2
(C) 1
(D) 4

129. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\left|\vec{a}+\vec{b}\right|+\left|\vec{a}-\vec{b}\right|$ is

(A) 2
(B) 4
(C) $2\sqrt{2}$
(D) $\sqrt{2}$

130. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. Then T maps the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) into a

(A) rectangle
(B) trapezium
(C) square
(D) parallelogram

131. The order of [7] in $(Z_9, +_9)$ is

(A) 9
(B) 6
(C) 3
(D) 4

132. Let $A = \{x \in R : |x - 1| + |x - 2| < 3\}$. Then A is

(A) open
(B) close
(C) both open and closed
(D) neither open nor closed

133. Let X be the set of all polynomials of degree $k \geq 1$ with integer coefficients. Then X is

(A) infinite
(B) uncountable
(C) countable
(D) finite

134. $\int \frac{\cos x}{z^3} dz$ is equal to

(A) πi
(B) $-\pi i$
(C) $2\pi i$
(D) $-2\pi i$

135. $\int \frac{dz}{z+2}$ is equal to

(A) $\frac{\pi}{2}$

(B) πi

(C) 1

(D) 0

136. The characteristic roots of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are

(A) 5, 1, 1
(B) 5, 2, 2
(C) 2, 2, 2
(D) 5,–1,–1

137. The radius of convergence of the series $\sum \frac{1}{n^p} z^n$ is

(A) 0
(B) 1
(C) 2
(D) ∞

138. The function $f(z) = \log z$ is

(A) everywhere analytic
(B) nowhere analytic
(C) not analytic at $z = 1$
(D) not analytic at $z = 0$

139. The value of the integral $\int_C \frac{z^2+3}{z-2}\,dz$ where C is the circle at centre 0 and of radius $3/4$, is

(A) 0
(B) 2
(C) πi
(D) $2\pi i$

140. The vector space of dimension 2 is

(A) $\mathbb{R}\times\mathbb{R}$ over $\mathbb{Q}$
(B) $\mathbb{C}\times\mathbb{C}$ over $\mathbb{R}$
(C) $\mathbb{C}\times\mathbb{C}$ over $\mathbb{Q}$
(D) $\mathbb{Q}\times\mathbb{Q}$ over $\mathbb{Q}$

141. If $T:\mathbb{R}^2 \to \mathbb{R}$ is the linear transformation for which $T\left((1,1)\right) = 5,\ T\left((0,1)\right) = -3,$ then $T\left((a,b)\right)$ is

(A) $5a-3b$
(B) $-3a-5b$
(C) $8a-3b$
(D) $3a+8b$

142. Let $f:\mathbb{R} \to \mathbb{R}$ be a polynomial and let $\left(x_n\right)$ be a sequence of real numbers converging to 2. Then the sequence $\left(f\left(x_n\right)\right)$

(A) does not converge
(B) converges to $f(2)$
(C) is not bounded
(D) converges to 2

143. If $f(x) = x^2$ for all $x \in \mathbb{R}$, then f is

(A) not continuous on $\mathbb{R}$
(B) uniformly continuous on $\mathbb{R}$
(C) not uniformly continuous on $\mathbb{R}$
(D) None of the above

144. $\lim_{x \to 0} \frac{e^x - x - 1}{x^2} =$

(A) 0
(B) $\frac{1}{2}$
(C) 2
(D) $\frac{1}{4}$

145. If $Z = f(y-z, z-x, x-y)$, then $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial z}$ is

(A) 1
(B) –1
(C) 2
(D) 0

146. The conjugate harmonic function of $u(x,u) = x^2 - y^2$ is

(A) $2xy + c$
(B) $2xy + y$
(C) $xy + c$
(D) $xy - c$

147. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ is

(A) 0
(B) ∞
(C) 1
(D) 4

148. For the function $f(z)=\frac{z-\sin z}{z^3}$, the point $z=0$ is

(A) a pole of order 2
(B) a removable singularity
(C) an essential singularity
(D) an isolated singularity

149. If D is the region bounded by the straight lines $y=x,\ y=0$ and $x=1,$ then the value of $\iint_D e^{\frac{y}{x}}\,dx\,dy$ is

(A) $\frac{1}{2}(e+1)$

(B) $\frac{1}{2}\left(e^2+1\right)$

(C) $\frac{1}{2}(e-1)$

(D) $\frac{1}{2}\left(e^2-1\right)$

150. The particular integral of the equation $\left(D^2-1\right)y=e^x+\cos 2x$ is

(A) $\frac{xe^x}{2}-\frac{\cos 2x}{5}$

(B) $-\frac{xe^x}{2}-\frac{\cos 2x}{5}$

(C) $\frac{xe^x}{2}+\frac{\cos 2x}{5}$

(D) $-\frac{xe^x}{2}+\frac{\cos 2x}{5}$

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ANSWER KEY

Subject Name: 612 MATHEMATICS

SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key
1	A	31	B	61	C	91	A	121	A
2	D	32	A	62	A	92	C	122	A
3	B	33	B	63	A	93	A	123	D
4	A	34	B	64	D	94	D	124	B
5	B	35	A	65	B	95	C	125	C
6	C	36	B	66	D	96	A	126	B
7	A	37	B	67	C	97	D	127	B
8	B	38	D	68	A	98	C	128	C
9	D	39	B	69	B	99	A	129	C
10	C	40	D	70	B	100	B	130	A
11	D	41	D	71	A	101	C	131	A
12	C	42	B	72	B	102	D	132	A
13	B	43	D	73	D	103	A	133	C
14	A	44	A	74	C	104	C	134	B
15	A	45	C	75	D	105	B	135	D
16	D	46	D	76	A	106	C	136	A
17	D	47	C	77	D	107	D	137	B
18	B	48	B	78	A	108	B	138	D
19	B	49	B	79	A	109	D	139	A
20	B	50	B	80	C	110	A	140	D
21	B	51	D	81	B	111	A	141	C
22	B	52	D	82	C	112	D	142	B
23	A	53	D	83	C	113	A	143	C
24	D	54	B	84	C	114	A	144	B
25	C	55	B	85	D	115	D	145	D
26	B	56	C	86	A	116	D	146	A
27	C	57	D	87	A	117	B	147	C
28	B	58	B	88	B	118	A	148	B
29	C	59	A	89	C	119	B	149	C
30	D	60	A	90	B	120	D	150	A