CAT - 2019 MATHEMATICS PG

1. Let $A = (a_{ij})$ be a matrix of order $m \times n$, where $a_{ij} = 1$ for all i, j. Then $rank(A)$ is

(A) m
(B) $m - n$
(C) 1
(D) 0

2. The vectors $(a, b, 0)$, $(1, 0, C)$ and $(1, 1, 0)$ are linearly independent in R^3 if

(A) $c \neq 0$ and $a = b$

(B) $c \neq 0$ and $a \neq b$

(C) $a \neq b$

(D) $a = b = c = 0$

3. The number of non-trivial subspaces of $\mathbb{R}^3$ over $\mathbb{R}$ is

(A) 0
(B) infinite
(C) 3
(D) 6

4. If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then

(A) $x > 3$

(B) $x < 4$

(C) $3 < x < 4$

(D) $\frac{7}{2}$

5. The area enclosed within the curve $|x| + |y| = 1$ is

(A) 2 sq units
(B) 4 sq units
(C) 6 sq units
(D) 8 sq units

6. If the point P(4, 3) is shifted by a distance $\sqrt{2}$ unit parallel to the line $y = x$, then the coordinates of P in the new position is

(A) (–5, –4)

(B) $(5 + \sqrt{2}, 4 + \sqrt{2})$

(C) $(5 - \sqrt{2}, 4 - \sqrt{2})$

(D) (5, 4)

7. If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is

(A) 1
(B) 2
(C) 4
(D) 6

8. The locus of the centers of the circles which touch both the axes is given by

(A) $x^2 - y^2 = 0$

(B) $x^2 + y^2 = 0$

(C) $x^2 - y^2 = 1$

(D) $x^2 + y^2 = 1$

9. If $3^{x+1} = 6^{\log_2 3}$, then x is

(A) 3
(B) 2
(C) $\log_3 2$
(D) $\log_2 3$

10. If the fourth roots of unity are z_1, z_2, z_3, z_4 then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ is equal to

(A) 1

(B) 0

(C) i

(D) $-i$

11. The equation $|z+1-i| = |z+i-1|$ represents a

(A) pair of straight lines
(B) circle
(C) parabola
(D) hyperbola

12. The radius of the circle $\left|\frac{z-i}{z+i}\right| = 5$ is equal to

(A) $\frac{13}{12}$

(B) $\frac{5}{12}$

(C) 5
(D) 625

13. The sum of the series $(1 + 2) + (1 + 2 + 2^2) + (1 + 2 + 2^2 + 2^3) + \ldots$ up to n terms is

(A) $2^{n+2} - n - 4$

(B) $2(2^n - 1) - n$

(C) $2^{n+1} - n$

(D) $2^{n+1} - 1$

14. $\int \frac{x + \sin x}{1 + \cos x} dx$ is equal to

(A) $x \tan\frac{x}{2} + c$

(B) $\log(1 + \cos x) + c$

(C) $x \cot\frac{x}{2} + c$

(D) $\log(x + \sin x) + c$

15. The solution of the differential equation $\cos x\,dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2}$, is

(A) $\sec x = \tan x + c$

(B) $y\sec x = \tan x + c$

(C) $y\tan x = \sec x + c$

(D) $\tan x = (\sec x + c)y$

16. The integrating factor of the differential equation $(y\log y)dx = (\log y - x)dy$ is

(A) $\frac{1}{\log y}$

(B) $\log(\log y)$

(C) $1 + \log y$

(D) $\log y$

17. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) 0
(B) 1
(C) 2
(D) 3

18. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar mutually perpendicular unit vectors, then $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ is

(A) 2
(B) 0
(C) 1
(D) 3

19. If $x^y y^x = 100$, then $\frac{dy}{dx}$ is equal to

(A) $-\frac{y(x+y)\log x}{x(x\log y + y)}$

(B) $-\frac{y(y+x\log x}{x(y\log x + x)}$

(C) $-\frac{y}{x}$

(D) $-\frac{x}{y}$

20. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is

(A) $\frac{y-1}{x+1}$

(B) $\frac{y+1}{x-1}$

(C) $\frac{x-1}{y-1}$

(D) $\frac{x-1}{y+1}$

21. If $x^x y^y z^z = c$, then $\frac{\partial z}{\partial x}$ is equal to

(A) $\frac{1+\log x}{1+\log z}$

(B) $-\frac{1+\log x}{1+\log z}$

(C) $\dfrac{1+\log z}{1+\log x}$

(D) $\dfrac{1-\log z}{1-\log x}$

22. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=x-[x]-\dfrac{1}{2}$ for $x\in\mathbb{R}$ where $[x]$ is the greatest integer not exceeding x, then the set $\left\{x\in\mathbb{R}: f(x)=\dfrac{1}{2}\right\}$ is equal to

(A) $\mathbb{Z}$, the set of all integers

(B) $\mathbb{N}$, the set of all natural numbers

(C) ϕ, the empty set

(D) $\mathbb{R}$, the set of all real numbers

23. The value of $\lim \log(\sin x)^{\tan x}$ is

(A) 1

(B) 1

(C) 0

(D) [illegible]

24. The value of $\lim_{x\to 0}\dfrac{5^x-5^{-x}}{2x}$ is

(A) log 5
(B) 0
(C) 1
(D) 2log 5

25. If $f(x)=2x^3+9x^2+\lambda x+20$ is decreasing function of x in the largest possible interval $(-2,-1)$, then the value of λ is equal to

(A) 12

(B) - 12

(C) 6

(D) - 6

26. The maximum sum of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + 18 + \ldots$ is

(A) 310

(B) 290

(C) 320

(D) 360

27. The product $(32)^{(32)^{\frac{1}{6}}(32)^{\frac{1}{36}}\ldots\infty}$ is equal to

(A) 16
(B) 64
(C) 32
(D) 0

28. The sum of 20 terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$ is equal to

(A) $210\sqrt{2}$

(B) $220\sqrt{2}$

(C) $300\sqrt{2}$

(D) $320\sqrt{2}$

29. If $x^2 - 3x + 2$ is one of the factors of the expression $x^4 - px^2 + q$, then

(A) $p = 4$, $q = 5$

(B) $p = -5, q = -4$

(C) $p = 5, q = 4$

(D) $p = -5, q = 4$

30. If $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then the value of $x^3 - 6x^2 + 6x$ is

(A) 3
(B) 2
(C) 1
(D) 4

31. If $\frac{{}^nP_{r-1}}{a} = \frac{{}^nP_r}{b} = \frac{{}^nP_{r+1}}{c}$, then

(A) $\sum \frac{1}{a} = 1$

(B) $abc = 1$

(C) $b^2 = a(b + C)$.

(D) $a^2 = c(a + B)$.

32. The number of numbers greater than 1000 but not greater than 4000 that can be formed with the digit $0, 1, 2, 3, 4,$ repetition of digits being allowed, is

(A) 374
(B) 375
(C) 376
(D) 377

33. The number of divisors of the form $4n + 2 (n \geq 0)$ of the integer 240 is

(A) 4
(B) 8
(C) 10
(D) 3

34. The coefficient of x^6 in $(1+x)^6 + (1+x)^7 + ... + (1+x)^{15}$ is

(A) $^{16}C_9$

(B) $^{16}C_5 - {}^{6}C_5$

(C) $^{16}C_6 - 1$

(D) $^{16}C_6 - {}^{6}C_5$

35. The solution set of the equation determinant of the matrix $\begin{bmatrix} 2 & 3 & x \\ 2 & 1 & x^2 \\ 6 & 7 & 3 \end{bmatrix} = 0$ is

(A) ϕ
(B) {0, 1}
(C) {1, –1}
(D) {1, –3}

36. The system of linear equations $x_1 + 2x_2 + x_3 = 3,\ 2x_1 + 3x_2 + x_3 = 3$ and $3x_1 + 5x_2 + 2x_3 = 1$ has

(A) infinite number of solutions
(B) exactly 3 solutions
(C) a unique solution
(D) no solution

37. If the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is commutative with the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then

(A) $a = 0, b = c$

(B) $b = 0, c = d$

(C) $c = 0, d = a$

(D) $d = 0, a = b$

38. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\alpha & \sin^2\beta \end{bmatrix}$ are two matrices such that *AB* is a null matrix, then $\alpha - \beta$ is

(A) 0

(B) an odd multiple of [illegible]

(C) an even multiple of $\frac{\pi}{2}$

(D) [illegible] = [illegible]

39. The value of the sum $\sum_{k=1}^{\infty}\frac{1}{k!}\sum_{n=1}^{k}2^{n-1}$ is equal to

(A) e
(B) $e^2 + e$
(C) e^2
(D) $e^2 - e$

40. The sum of the series $\cos x - \frac{1}{2}\cos^2 x + \frac{1}{3}\cos^3 x - \frac{1}{4}\cos^4 x + \ldots$ is

(A) $\log 2 + \log\left(\cos\left(\frac{x}{2}\right)\right)$

(B) $\log 2 - 2\log\left(\cos\left(\frac{x}{2}\right)\right)$

(C) $\log 2 - \log\left(\cos\left(\frac{x}{2}\right)\right)$

(D) $\log 2 + 2\log\cos\left(\frac{x}{2}\right)$

41. Let $A = \{x \in C : x^2 = 1\}$ and $B = \{x \in C : x^4 = 1\}$. Then the set $A \Delta B$ is equal to

(A) $\{1, -1\}$

(B) $\{1, -1, i, -i\}$

(C) $\{i, -i\}$

(D) [illegible]

42. If R is a relation over the set all real numbers and it is defined by $m \geq 0$, then R is

(A) reflexive and transitive
(B) reflexive and symmetric
(C) symmetric and transitive
(D) an equivalence relation

43. Let $f : \mathbb{R} \to \mathbb{R}$ be the mapping defined by $f(x) = x^3 + 1$. Then f is

(A) bijective
(B) surjective
(C) injective
(D) automorphism

44. Let $a, b, c \in N$ and b, c be coprime. If $aN = \{an : n \in N\}$ and $bN \cap cN = dN$, then

(A) $b = cd$

(B) $c = bd$

(C) $d = bc$

(D) $a = bc$

45. The number of onto mappings from the set $X = \{1, 2, \ldots, m\}$ in to the set $Y = \{1, 2\}$ is

(A) $2^m - 2$
(B) 2^m
(C) $2^{m-1} - 2$
(D) $2m$

46. If the probability of a defective bolt is 0.1, then the mean and the standard deviation of distribution of bolts in a total of 400 are

(A) 30, 3
(B) 40, 5
(C) 30, 4
(D) 40, 6

47. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is

(A) $\frac{1}{2}$

(B) $\frac{5}{9}$

(C) $\frac{4}{9}$

(D) $\frac{2}{3}$

48. Seven balls are drawn simultaneously from a bag containing 5 white and 6 green balls. The probability of drawing 3 white and 4 green balls is

(A) $\frac{7}{{}^{11}C_7}$

(B) $\frac{{}^{5}C_3 \cdot {}^{6}C_4}{{}^{11}C_7}$

(C) $\frac{{}^{5}C_3 \; {}^{6}C_2}{{}^{11}C_7}$

(D) $\frac{{}^{6}C_3 \; {}^{5}C_4}{{}^{11}C_7}$

49. A biased coin with probability p, $(0 < p < 1)$ of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is $\frac{2}{5}$, then p is equal to

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

50. Which of the following numbers is rational?

(A) $\sin 15^\circ$

(B) $\cos 15^\circ$

(C) $\sin 15^\circ \cos 15^\circ$

(D) $\sin 15^\circ \cos 75^\circ$

51. The equation $\sqrt{3}\sin x + \cos x = 4$ has

(A) only one solution
(B) two solutions
(C) infinitely many solutions
(D) no solutions

52. The equation $x^2 + y^2 + 4x + 6y + 13 = 0$ represents

(A) circle
(B) a pair of two distinct straight line
(C) a point
(D) a pair of coincident straight line

53. In the interval $(-3,3)$, the function $f(x)=\frac{x}{3}+\frac{3}{x}, x\neq 0$ is

(A) increasing
(B) decreasing
(C) neither increasing nor decreasing
(D) partly increasing and partly decreasing

54. Let *G* be a group of even order with identity element *e*. Then

(A) $a^2=e$ for some $a\in G$

(B) $a^3=e$ for some $a\in G$

(C) $a^{O(G)}=e$ for some $a\in G$

(D) $a^{O(G)}=e$ for no $a\in g$

55. If every element of a group *G* is its own inverse, then *G* is

(A) abelian
(B) infinite
(C) cyclic
(D) finite

56. The set of congruent 8 classes $[1],[3],[5],[7]$ under multiplication modulo 8 forms

(A) a cyclic group
(B) a monoid
(C) an abelian group
(D) the Klein four group

57. Consider the group $(Q^+,*)$ where Q^* is the set of all positive relational numbers and * is defined by *a* * *b* = $\frac{ab}{2}, a,b\in Q^+$. Then the inverse of 3 is

(A) 1/3

(B) 3/4

(C) 4/3

(D) - 3

58. The generators of the cyclic group $G = \{8^n \mid n \in Z\}$ are

(A) 2 and $\frac{1}{2}$

(B) 4 and $\frac{1}{4}$

(C) 6 and $\frac{1}{6}$

(D) 8 and $\frac{1}{8}$

59. The number of points on the circle $3x^2 + 2y^2 - 3x = 0$ which are at distance 2 from the point (–2, 1) is

(A) 2
(B) 0
(C) 1
(D) 3

60. Let $t_r = 2^{\frac{r}{2}} + 2^{\frac{-r}{2}}$. Then $\sum_{r=1}^{10} t_r^2$ is equal to

(A) $\frac{2^{21} - 1}{2^{10}} + 20$

(B) $\frac{2^{21} - 1}{2^{10}} + 19$

(C) $\frac{2^{21} - 1}{2^{10}} - 1$

(D) 1

61. If $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ and a, b, c are in geometrical progression, then x, y, z are in

(A) AP

(B) GP

(C) HP

(D) $a = b = c$

62. The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. The number of sides of the polygon is

(A) 9
(B) 10
(C) 16
(D) 5

63. If $x + \frac{1}{x} = 5,$ then $\left(x^3 + \frac{1}{x^3}\right) - 5\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right)$ is equal to

(A) 0

(B) 5

(C) -5

(D) 10

64. If roots of the equation $x^n - 1 = 0$ are $1, a_1, a_2, \ldots, a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3)\ldots(1 - a_{n-1})$ is

(A) n
(B) n
(C) n^n
(D) 0

65. If $ax^2 + bx + 10 = 0$ does not have two distinct real roots, then the least value of $5a + b$ is

(A) -3

(B) -2

(C) 3
(D) 2

66. The roots of the equation $x^{\sqrt{x}} = \sqrt{x^{x}}$ are

(A) 0 and 4

(B) 0 and 1

(C) -1 and 4

(D) 1 and 4

67. The total number of 9 digit numbers with different digits is

(A) 10!
(B) 9!
(C) 9.9!
(D) 10.10!

68. If a, b, c are three natural numbers which are in AP and $a + b + c = 21$, then the possible number of values of the ordered triplet (a, b, c) is

(A) 15
(B) 14
(C) 13
(D) 12

69. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

(A) 2
(B) 1
(C) 0
(D) 8

70. In the expansion of $\left(x - \frac{1}{x}\right)^6$ the constant term is

(A) 20

(B) - 20

(C) 30

(D) - 30

71. For $|x| < 1$, the constant term in the expansion of $\frac{1}{(x-1)^2(x-1)}$ is

(A) 2

(B) 1

(C) 0

(D) $\frac{-1}{2}$

72. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then

(A) $x = 39$

(B) $x = 63$

(C) $36 \le x \le 63$

(D) $x \ne 39$

73. If $P(A) = 0.65, P(B) = 0.80,$ then $P(A \cap B)$ lies in the interval

(A) [0.30, 0.80]
(B) [0.35, 0.75]
(C) [0.4, 0.70]
(D) [0.45, 0.65]

74. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ is equal to

(A) 0

(B) ab

(C) $\frac{b}{a}$

(D) $\frac{a}{b}$

75. The value of $\frac{1}{\cos 80^\circ} - \frac{\sqrt{3}}{\sin 80^\circ}$ is equal to

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 2
(D) 4

76. If $y = f(x^3)$, $z = g(x^5)$, $f'(x) = \tan x$ and $g'(x) = \sec x$, then the value of $\frac{dy}{dz}$ is

(A) $\frac{3}{5x^2} \cdot \frac{\tan x^3}{\sec x^5}$

(B) $\frac{5x^2}{3} \cdot \frac{\sec x^5}{\tan x^3}$

(C) $\frac{3x^2}{5} \cdot \frac{\tan x^3}{\sec x^5}$

(D) $\frac{5}{3x^2} \cdot \frac{\sec x^5}{\tan x^3}$

77. If $\sqrt{x} + \sqrt{y} = 4$, then $\frac{dx}{dy}$ at $y = 1$ is

(A) -1

(B) -3

(C) 3

(D) 1

78. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\infty}}}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y + x}{y^2 - 2x}$

(B) $\frac{y^3 - x}{2y^2 - 2xy - 1}$

(C) $\frac{y^3 + x}{2y^2 - x}$

(D) $y^2 + x^2$

79. The general solution of the differential equation $y(x^2 y + e^x)dx - e^x dy = 0$ is

(A) $x^3y - 3e^x = cy$

(B) $x^3y + 3e^x = 3cy$

(C) $y^3x - 3e^y = cx$

(D) $y^3x + 3e^y = cx$

80. For the operation * defined by $a*b = \frac{ab}{2}$, the identity element is

(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

81. If W_1 and W_2 are finite dimensional subspaces with the same dimension and $W_1 \subseteq W_2$, then

(A) $W_1 - W_2 = \phi$

(B) $W_1 \cap W_2 = W_1$

(C) $W_1 \cap W_2 \subset W_1$

(D) $W_1 = W_2$

82. The basis of $\mathbb{R}^3(\mathbb{R})$ from the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ where $\alpha_1 = (1, -3, 2), \alpha_2 = (2, 4, 1)$, $\alpha_3 = (3, 1, 3)$ and $\alpha_4 = (1, 1, 1)$ is

(A) $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$

(B) $\{\alpha_1, \alpha_3\}$

(C) $\{\alpha_1, \alpha_2, \alpha_4\}$

(D) $\{\alpha_2, \alpha_3, \alpha_4\}$

83. Let $S(\mathbb{R})$ be the vector space of all polynomial functions $f(x)$ with coefficients as elements of the field $\mathbb{R}$ of real numbers. Let D and T be two linear operators on V defined by $D(f(x)) = \frac{d}{dx} f(x)$ and $T(f(x)) = \int_0^x f(x)\,dx$ for every $f(x) \in V$. Then

(A) $TD = I$

(B) $DT = I$ and $TD \neq I$

(C) $DT \neq I$

(D) $TD = I$ and $DT \neq I$

84. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. Then T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into a

(A) rectangle
(B) trapezium
(C) square
(D) parallelogram

85. Let T be a linear operator on $V_3(\mathbb{R})$ defined by $T(a,b,c) = (3a, a - b, 2a + b + c)$ for all $(a,b,c) \in V_3(\mathbb{R})$. Then $T^{-1}(p,q,r) =$

(A) $\left(\frac{1}{3p}, \frac{1}{p-q}, \frac{1}{2p+q+r}\right)$

(B) $\left(\frac{p}{3}, \frac{p}{3} - q, r - p + q\right)$

(C) $\left(\frac{p}{3} + q, \frac{1}{p-q}, 2r - p + \frac{1}{q}\right)$

(D) $\left(\frac{p}{3}, q + 2p, r - p - q\right)$

86. A vector of unit length which is orthogonal to the vector $\alpha = (2, -1, 6)$ of $V_3(\mathbb{R})$ with respect to standard inner product is

(A) $(2, 2, -1)$

(B) $\left(\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$

(C) $\left(\frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(D) $\left(\sqrt{2}, -\sqrt{2}, \frac{-1}{\sqrt{2}}\right)$

87. Let V be the vector space $V_2(\mathbb{C})$ with the standard inner product. Let T be the linear operator defined by $T(1,0) = (1, -2),\ T(0,1) = (i, -1)$. Then its adjoint T^* is

(A) $T^*(a,b) = (a + b, a + ib)$

(B) $T^*(a,b) = (a - 2b, a - ib)$

(C) $T^*(a,b) = (a - 2b, -ia - b)$

(D) $T^*(a,b) = (a - 2b, ia - b)$

88. The value of $\bar{r}$ satisfying $\frac{d^2\bar{r}}{dt^2} = \bar{a}t + \bar{b}$, where $\bar{a}$ and $\bar{b}$ are constant vectors, given that when $t = 0,\ \bar{r} = 0$ and $\frac{d\bar{r}}{dt} = u$ is

(A) $\bar{r} = \frac{1}{2}at^3 + \frac{1}{6}bt^2 + ut$

(B) $\bar{r} = \frac{1}{6}at^3 + \frac{1}{2}bt^2 + ut$

(C) $\bar{r} = \frac{1}{2}at^3 + ubt$

(D) $\bar{r} = \frac{1}{2}at^3 + ubt^2 + ut$

89. The value of $div(r^n\bar{r})$ is

(A) $(n+3)r^n$

(B) $(n+3)r^{-2n}$

(C) $(n+3)r^2$

(D) $nr^{-(n+3)}$

90. Green's theorem applied to $\int_C (\cos x \sin y - xy)dx + \sin x \cos y dy,$ where C is the circle $x^2 + y^2 = 1$, yields

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 0

(D) $\frac{2}{3}$

91. The length of the space curve $\overline{x}(t)$ over the parameter range $a \le t \le b$ can be computed by

(A) integrating the norm of its tangent vector

(B) integrating the square of the norm of its tangent vector

(C) integrating the square of the norm of $\overline{x}(t)$

(D) integrating the square root of the norm of $\overline{x}(t)$

92. The sum $\cos\theta + \cos 3\theta + \ldots + \cos(2n+1)\theta$ is equal to

(A) $\frac{\sin(2n+2)\theta}{2\sin\theta}$

(B) $\frac{\cos(2n+1)\theta}{\cos\theta}$

(C) $\frac{\cos(2n+2)\theta}{2\cos\theta}$

(D) $\frac{\sin(2n+1)\theta}{\sin\theta}$

93. The real and imaginary parts of $(-i)^i$ are respectively

(A) $e^{\pi+2n\pi}, -1$ for $n \in z$

(B) $e^{\frac{\pi}{2}+n\pi}, \log|i|$ for $n \in z$

(C) $e^{\pi-\frac{n}{2}\pi}, \log|-i|$ for $n \in z$

(D) $e^{\frac{\pi}{2}-2n\pi}, 0$ for $n \in z$

94. The singularities of $f(z) = \frac{1}{z\sin z}$ are

(A) simple poles at $z = n\pi (n = 0, \pm 1, \pm 2, ...)$

(B) simple poles at $z = n\pi (\pm 1, \pm 2, ...)$ and double pole at $z = 0$

(C) removable singularity at $z = n\pi (\pm 1, \pm 2, ...)$ and essential singularity at $z = 0$

(D) essential singularity at $z = 0$ and simple poles at $z = n\pi (\pm 1, \pm 2, ...)$

95. If $f(z) = z^2$ and $\gamma = e^{it}$, $t \in [0, \pi]$, then $\oint_\gamma f(z)\,dz$ is equal to

(A) 2/3

(B) $-2/3$

(C) 1/3

(D) 1/2

96. The value of $\int_C \frac{e^z}{\pi i - 2z} dz$, where [redacted] is a circle with center at 0 and radius 2, is

(A) $-\pi i$

(B) $-\pi + i$

(C) π

(D) $\pi^2 - i$

97. The set $A = \left\{1, -1, \frac{1}{2}, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{3}, \ldots\right\}$ is

(A) open

(B) closed

(C) both open and closed

(D) neither open nor closed

98. The value of $\lim_{x \to 0} \frac{e^{1/x}}{e^{1/x} + 1}$

(A) is 1

(B) is □

(C) is 0

(D) does not exist

99. The function $f(x) = \frac{x - |x|}{x}$ is

(A) differentiable

(B) continuous everywhere except zero

(C) continuous for $x > 0$ alone

(D) continuous for $x \geq 0$ alone

100. The function $f(x) = \begin{cases} 2x, & 0 \leq x < 1 \\ 3, & x = 1 \\ 4x, & 1 < x \leq 2 \end{cases}$

(A) is continuous
(B) has a discontinuity of the first kind at $x = 1$
(C) has a discontinuity of the first kind from left at $x = 1$
(D) has a discontinuity of the second kind at $x = 1$

101. The integral $\int_0^1 \frac{1}{x} dx$

(A) converges absolutely
(B) converges monotonically
(C) converges conditionally
(D) diverges

102. The function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

(A) discontinuous at $x = 0$
(B) not differentiable at $x = 0$
(C) differentiable everywhere but its derivative is not continuous at $x = 0$
(D) not differentiable at $x = 0$ and its derivative is also not differentiable at $x = 0$

103. The Fourier series corresponding to $f(x) = \begin{cases} 0, & -\pi \leq x < 0 \\ 1, & 0 \leq x \leq \pi \end{cases}$ is

(A) $\frac{1}{4}+\frac{1}{\pi}\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\ldots\right)$

(B) $\frac{1}{4}+\frac{1}{\pi}\left(\cos x-\frac{\cos 3x}{3}+\frac{\cos 5x}{5}+\ldots\right)$

(C) $\frac{1}{2}+\frac{2}{\pi}\left(\sin x+\frac{\sin 2x}{2}+\frac{\sin 3x}{3}+\ldots+\cos x+\frac{\cos 2x}{2}+\frac{\cos 3x}{3}+\ldots\right)$

(D) $\frac{1}{2}+\frac{2}{\pi}\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\ldots\right)$

104. If three coplanar forces keep a rigid body in equilibrium, then

(A) they are all concurrent
(B) they are all parallel
(C) either they are all parallel or concurrent
(D) they all act along the sides of a triangle in order

105. The shape of a uniform string hanging under gravity is given by

(A) $y = c\cosh\left(\frac{x}{L}\right)$

(B) $x = r\cos^{-1}\left(1 - \frac{y}{r}\right) - \sqrt{y(2r - y)}$

(C) $(x + c)^2 = 4ay$

(D) $y = c\cosh^2\left(\frac{x}{L}\right)$

106. The period of a simple pendulum is

(A) $2\pi\sqrt{\frac{g}{l}}$

(B) $4\pi\sqrt{\frac{g}{l}}$

(C) $2\pi\sqrt{\frac{l}{g}}$

(D) $2\pi\sqrt{\frac{l^2}{g}}$

107. The moment of inertia of a right circular hollow cylinder of base radius a and mass M about the axis of the cylinder is

(A) Ma^3
(B) Ma^2
(C) $Ma^2/3$
(D) $Ma^3/2$

108. The distance of the point (1, −2, 3) from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, − 6 is

(A) $\sqrt{7}$

(B) $\sqrt{7}/5$

(C) 3
(D) 1

109. The planes $x - 2y + z - 3 = 0,\ x + y - 2z - 3 = 0$ and [illegible] $- z - [illegible] = 0$

(A) intersect at a point
(B) intersect along a line
(C) do not intersect at all
(D) form a triangular prism

110. The plane $x + 2y - z = 4$ and the sphere $x^2 + y^2 + z^2 + [illegible] + z - 2 = 0$

(A) do not meet each other
(B) intersect at only one point
(C) intersect along a circle of unit radius
(D) intersect along the great circle

111. The equations of two tangent planes to the sphere $x^2 + y^2 + z^2 - 2y - 6z + 5 = 0$ which are parallel to the plane $2x + 2y - z = 0$ are

(A) $6x + 6y - 3z + (1 \pm 2\sqrt{5}) = 0$

(B) $2x + 2y - z + (1 \pm 3\sqrt{5}) = 0$

(C) $6x + 6y - 3z + 3\sqrt{5} = 0,\ 2x + 2y - z + 3\sqrt{5} = 0$

(D) $6x + 6y - 3z + (1 + 3\sqrt{5}) = 0,\ 2x + 2y - z + (1 - 3\sqrt{5}) = 0$

112. The equation of a right circular cone with vertex at origin 0, axis the x-axis and semi-vertical angle ✔ is

(A) $x^2 + y^2 = x^2 \sec h^2 \alpha$

(B) $y^2 + z^2 = x^2 \tanh^2 \alpha$

(C) $y^2 + z^2 = x^2 \tan^2 \alpha$

(D) $x^2 + y^2 = x^2 \tanh^2 \alpha$

113. The equation of a cylinder which passes through $f_1(x,y,z) = 0,\ f_2(x,y,z) = 0$ and having its generator parallel to the x-axis can be obtained by

(A) adding f_1 and f_2

(B) adding f_1 with λf_1 where λ is a scalar

(C) eliminating *x*

(D) multiplying f_1 and f_2

114. If the equation $M(x,y)dx + N(x,y)dy = 0$ is exact, then

(A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(B) $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$

(C) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(D) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

115. The solution of $D^2(D^2 + 4)y = 96x^2$ is

(A) $y = \frac{1}{4}\left(x^2 - \frac{1}{2}\right) + A\cos 2x + B\sin 2x$

(B) $y = 2x^4 - 6x^2 + A\cos 2x + B\sin 2x + E + Fx^2$

(C) $y = 2x^4 - 4x^2 + A\cos 2x + B\sin 2x + E + Fx^2$

(D) $y = \frac{1}{4}\left(x^2 - \frac{1}{2}\right) + A\cos 2x + B\sin 2x + E + Fx^2$

116. The differential equation obtained by eliminating a, b and c from $z = a(x+y) + b(x-y) + abt + c$ is

(A) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4\frac{\partial^2 z}{\partial t^2}$

(B) $\frac{\partial^2 z}{\partial x^2} - \left(\frac{\partial z}{\partial t}\right)^2 = \frac{\partial^2 z}{\partial y^2}$

(C) $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial y} = 4\frac{\partial^2 z}{\partial t^2}$

(D) $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = 4\frac{\partial z}{\partial t}$

117. An envelope of $y = x + \frac{1}{4}(x-c)^2$ is

(A) $y^2 = 2x$

(B) $y = x^2 + 3x$

(C) $3y = x + 2$

(D) $y = x$

118. The Legendre equation is given by

(A) $x^2 y'' + xy' + (x^2 - n^2)y = 0$

(B) $(1-x^2)y'' + 2xy' - n(n+1)y = 0$

(C) $(1-x^2)y'' - 2xy' + n(n+1)y = 0$

(D) $(1-x^2)y'' - xy' + (x^2 - n^2)y = 0$

119. The harmonic function $\phi(x, y)$ cannot attain either its maximum or minimum inside a region Ω of $\mathbb{R}^2$

(A) unless ϕ is trivial
(B) unless ϕ is a constant function
(C) if ϕ is unbounded
(D) if ϕ and its first partial derivatives are unbounded

120. Let $S = \mathbb{R} - \{0,1\}$ and let f_1, f_2, f_3 be functions on S defined by $f_1(x) = x$, $f_2(x) = \dfrac{1}{1-x}$, $f_3(x) = \dfrac{x-1}{x}$. Then these functions under the operation composition of functions form a

(A) group
(B) semigroup
(C) abelian group
(D) monoid

121. The total number of subgroups of $\mathbb{Z}$ contained in $20\mathbb{Z}$ is

(A) 6
(B) 2
(C) infinite
(D) zero

122. Let G be the group of all 2 × 2 diagonal matrices under multiplication. Then the centre of G is

(A) $\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$

(B) $\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}i&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&i\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$

(C) $\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}-1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&-1\end{bmatrix}, \begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$

(D) G itself

123. A right inverse of matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}$

124. Let *R* be the ring of all real valued functions defined on $\mathbb{R}$, under pointwise addition and multiplication. Which of the following subset of *R* is not a subring?

(A) Set of all continuous functions
(B) Set of all polynomial functions
(C) Set of all functions which are zero at finitely many points together with the zero function
(D) Set of all functions which are zero at infinite number of points

125. If *A* and *B* are sets such that $O(A) = 5$ and $O(B) = 3$, then the number of binary relations from *A* to *B* is

(A) 2^{25}
(B) 2^{9}
(C) 2^{15}
(D) 2^{24}

CUSAT COMMON ADMISSION TEST 2019

126. Let $R = \{(a, a), (b, c), (a, b)\}$ be a relation on the set $\{a, b, c\}$. The minimum set of elements that should be added to R so that it becomes antisymmetric is

(A) $\{(c, b), (b, a)\}$

(B) $\{(b, b), (c, c), (c, b), (b, a), (a, c), (c, a)\}$

(C) $\{(a, c)\}$

(D) ϕ

127. Let the line $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$ lie on the plane $x + 3y - \alpha z + \beta = 0$. Then the point (α, β) lie in

(A) $x + y = 1$

(B) $x - y = 1$

(C) $x + y = 13$

(D) $x - y = 2$

128. The points on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$ at a distance 5 units from the point (1, 3, 3) are

(A) (7, 8, [illegible]) and (-3, -4, 2)
(B) (8, 7, [illegible]) and (-2, -3, -3)
(C) (3, 2, 2) and (-2, -1, 3)
(D) (-2, -1, 3) and (4, 3, 7)

129. Ram and Gopi appear for an interview for two vacancies in a company. The probability of Ram's selection is $\dfrac{1}{5}$ and that of Gopi is $\dfrac{1}{6}$. The probability that none of them is selected is

(A) $\dfrac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{30}$

130. The number of positive divisors of 50000 is

(A) 20
(B) 30
(C) 40
(D) 50

131. The slope of the line $\frac{1}{r} = \cos(\theta - \alpha) + \rho\cos\theta$ is

(A) $-\frac{\rho + \cos\alpha}{\sin\alpha}$

(B) $\frac{\sin\alpha}{\rho + \cos\alpha}$

(C) $\frac{\cos\alpha}{\rho + \sin\alpha}$

(D) $\frac{\rho - \sin\alpha\cos\alpha}{\sin\alpha}$

132. The angle between the lines $6x = 3y = 4x$ and $2x = -y = z$ is

(A) $\frac{\pi}{3}$

(B) 0

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

133. When a force $F = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ displaces a particle in the *xy* plane from (0, 0) to (1, 1) along the curve $y = x$, the work done is

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) 2

(D) $\frac{3}{2}$

134. The unit normal to the surface $x^3 - xyz + z^3 = 1$ at the point (1, 1, 1) is

(A) $2\hat{i} - \hat{j} + \hat{k}$

(B) $\frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$

(C) $\frac{2}{3}(2\hat{i} - \hat{j} + 2\hat{k})$

(D) $\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$

135. The directional derivative of $Q = xy + yz + zx$ at the point (1, 2, 3) along the *x*-axis is

(A) 4
(B) 3
(C) 7
(D) 5

136. The Laplace transform of $\frac{\sin wt}{t}$ is

(A) $\tan^{-1}\frac{s}{w}$

(B) $\cot^{-1}\frac{s}{w}$

(C) $\frac{1}{s}\frac{w}{s^2 + w^2}$

(D) $\frac{w}{s^2 + w^2}$

137. The equation of the sphere which has its centre at (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$ is

(A) $x^2 + y^2 + z^2 + 12x + 2y - 4z + 16 = 0$

(B) $x^2 + y^2 + z^2 - 12x + 2y + 4z + 16 = 0$

(C) $x^2 + y^2 + z^2 - 6x + 2y - 4z + 16 = 0$

(D) $x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

138. The value of the product $\left(1+\frac{1}{1!}+\frac{1}{2!}+...\right)\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+...\right)$ is

(A) 1

(B) e^2

(C) 0

(D) $\log_e 2$

139. The value of $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y+2z)\,dzdydz$ is

(A) $\frac{1}{53}$

(B) $\frac{2}{21}$

(C) $\frac{1}{6}$

(D) $\frac{5}{3}$

140. The matrix $\begin{bmatrix}\cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha\end{bmatrix}$ is unitary when α is

(A) $(2x+1)\frac{\pi}{2}$

(B) $(3x+1)\frac{\pi}{2}$

(C) $(4x+1)\frac{\pi}{2}$

(D) $(5x+1)\frac{\pi}{2}$

141. The largest eigenvalue of $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

(A) 16
(B) 21
(C) 48
(D) 64

142. Let $A = \begin{bmatrix} a & 1 & 4 \\ 0 & b & 7 \\ 0 & 0 & 3 \end{bmatrix}$ be a matrix with real entries. If the sum and product of the eigenvalues are 10 and 30 respectively, then $a^2 + b^2$ equals

(A) 20
(B) 40
(C) 58
(D) 65

143. A group G is generated by the elements *x*, *y* with the relations $x^3 = y^2 = (xy)^2 = 1$. Then the order of the group G is

(A) 4
(B) 6
(C) 8
(D) 12

144. The number of group homomorphisms from $\mathbb{Z}/20\mathbb{Z}$ to $\mathbb{Z}/29\mathbb{Z}$ is

(A) 1
(B) 20
(C) 29
(D) 25

145. Let $f : R \to [0, \infty)$ be continuous function such that $(f(x))^2$ is uniformly continuous. Then

(A) f is bounded
(B) f may not be uniformly continuous
(C) f is uniformly continuous
(D) f is unbounded

146. For each x in [0, 1], let $f(x) = x$ if x is rational and let $f(x) = 1 - x$ if x is irrational. Then

(A) $f(x+1) = f(x)$

(B) $f(x) - f(1-x) = 1$

(C) $f(x-1) - f(x) = 1$

(D) $f(x) + f(1-x) = 1$

147. The residue at $z = 3$ of $f(z) = \dfrac{z}{(z-1)^4(z-2)(z-3)}$ is

(A) $\frac{101}{16}$

(B) -8

(C) $\frac{27}{16}$

(D) 0

148. If i and $2i$ are two roots of a biquadratic equation, then the equation is

(A) $x^4 + 5x^2 + 4 = 0$

(B) $x^4 + x^2 + 4 = 0$

(C) $x^4 + 5x^2 - 4 = 0$

(D) $x^4 - 5x^2 + 4 = 0$

149. The differential equation of the curve $y = x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \ldots$ is

(A) $\frac{dy}{dx} = 2y$

(B) $\frac{d^2y}{dx^2} = y$

(C) $\frac{dy}{dx} = x^2 y$

(D) $\frac{dy}{dx} = 2xy$

150. The function $y(x) = cx + e^c$ is the general solution of the differential equation

(A) $\frac{dy}{dx} = xy + e^x$

(B) $\frac{dy}{dx} = xy + e^{y'}$

(C) $y - x\frac{dy}{dx} = e^{y'}$

(D) $\log\left(y - x\frac{dy}{dx}\right) = \frac{dy}{dx}$

MATHEMATICS PG - ANSWER KEY

TEST CODE: 612

QN. NO.	KEY	QN. NO.	KEY	QN. NO.	KEY	QN. NO.	KEY	QN. NO.	KEY
1	C	26	A	51	D	76	A	101	D
2	A	27	B	52	C	77	B	102	C
3	D	28	A	53	B	78	D	103	D
4	D	29	C	54	A	79	B	104	C
5	A	30	B	55	A	80	C	105	A
6	D	31	C	56	C	81	D	106	C
7	A	32	B	57	C	82	C	107	B
8	A	33	A	58	D	83	B	108	D
9	D	34	A	59	A	84	D	109	D
10	B	35	D	60	B	85	B	110	C
11	A	36	D	61	A	86	B	111	B
12	B	37	C	62	A	87	C	112	C
13	A	38	B	63	A	88	B	113	C
14	A	39	D	64	A	89	C	114	D
15	C	40	D	65	B	90	C	115	B
16	D	41	A	66	D	91	A	116	D
17	C	42	D	67	C	92	A	117	D
18	C	43	A	68	C	93	D	118	C
19	B	44	C	69	B	94	B	119	B
20	A	45	A	70	A	95	B	120	C
21	B	46	D	71	D	96	C	121	C
22	C	47	C	72	C	97	C	122	D
23	C	48	C	73	D	98	D	123	A
24	D	49	A	74	D	99	B	124	D
25	A	50	C	75	D	100	B	125	C

QN. NO.	KEY
126	D
127	A
128	D
129	A
130	B
131	A
132	D
133	B
134	B
135	D
136	B
137	D
138	A
139	B
140	A
141	B
142	C
143	B
144	A
145	C
146	D
147	C
148	A
149	D
150	C