61414

ROLL No. | | | | | |

TEST BOOKLET No. 176

TEST FOR POST GRADUATE PROGRAMMES

STATISTICS

Time: 2 Hours

Maximum Marks: 450

INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.
2. Write your Roll Number in the space provided on the top of **this page**.
3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the **Answer Sheet**. Darken the appropriate bubbles with a **Ball Point Pen.**
4. The paper consists of 150 objective type questions. All questions carry equal marks.
5. Each question has four alternative responses marked **A, B, C** and **D** and you have to **darken** the bubble fully by a **Ball Point Pen** corresponding to the correct response as indicated in the example shown on the Answer Sheet.
6. Each correct answer carries **3** marks and each wrong answer carries **1** minus mark.
7. Space for rough work is provided at the end of this Test Booklet.
8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.
9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of any such unforeseen happenings, the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

SEAL

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STATISTICS

1. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

   (A) 8
   (B) 6
   (C) ∞
   (D) 4

2. The value of the integral $\int_{-1}^{1} |x| \, dx$ is

   (A) 0
   (B) 1
   (C) 2
   (D) ∞

3. $\lim_{x \to 1} \frac{(x-1)\sqrt{x}}{\log x}$ is

   (A) 0
   (B) 2
   (C) 1
   (D) $\frac{1}{2}$

4. The rank of the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is

   (A) 4
   (B) 1
   (C) 2
   (D) 3

5. For a matrix A, if $A^2 = I$, then matrix A is called

   (A) Orthogonal
   (B) Idempotent
   (C) Nil potent
   (D) None of the above

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6. The inverse of the matrix $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 3 & 2 \\ -7 & -5 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} -3 & 2 \\ 7 & -5 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

7. The system of m equations $AX = b$ in n variables is consistent if and only if

(A) $\text{rank}(A) = \min(m, n)$

(B) $\text{rank}(A) \le m + n$

(C) $\text{rank}(A) \le \text{rank}(Ab)$

(D) $\text{rank}(A) = \text{rank}(Ab)$

8. The largest eigen value of $A = \begin{bmatrix} 8 & 12 \\ 2 & 6 \end{bmatrix}$ is

(A) 6

(B) 4

(C) 12

(D) 2

9. The quadratic form $q(x) = 12x_1^2 + 12x_1x_2 + 3x_2^2$ is

(A) positive definite

(B) positive semi-definite

(C) negative definite

(D) negative semi-definite

10. If $A^2 - A + 1 = 0$, then the inverse of A is

(A) A

(B) A^2

(C) $1 - A$

(D) $A - 1$

11. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is

(A) A is zero matrix

(B) $A = (-1)$ I where I is a unit matrix

(C) $A^2 = 1$

(D) A^{-1} does not exist

12. A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$. Then which of the following will be always true?

(A) $A = B$
(B) $AB = BA$
(C) either A or B is a zero matrix
(D) either A or B is an identity matrix

13. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4ac$ is equal to

(A) 2
(B) 1
(C) 3
(D) 4

14. The number of real solution of the equation $x^2 - 3|x| + 2 = 0$ is

(A) 4
(B) 1
(C) 2
(D) 3

15. If p and q are the roots of the equation $x^2 + px + q = 0$, then the value of p and q is

(A) $p = 2, q = 1$
(B) $p = 1, q = 2$
(C) $p = 1, q = -2$
(D) $p = -2, q = 1$

16. A r.v. X has the p.m.f. $f(x)=\left(\frac{1}{2}\right)^x$, $x=1,2,3....$ and $A=\{x:x=1,3,5,....\}$. Then $P(A)$ is

(A) 1/3
(B) 2/3
(C) 1/2
(D) 1/4

17. A r.v. X has the p.d.f $f(x)=\frac{1}{2}x^2e^{-x}$, $x>0$. Then $E(X)$ is

(A) 2
(B) 4
(C) 1
(D) 3

18. A r.v. X has the p.d.f $f(x)=4x^3$, $0<x<1$. The 20[th] percentile of the distribution is

(A) $(0.2)^4$
(B) $\sqrt[4]{0.20}$
(C) $\sqrt[2]{0.40}$
(D) $\sqrt[4]{0.80}$

19. A r.v. X has the p.d.f $f(x)=xe^{-x}$, $0<x<\infty$.
The m.g.f. of X, $M(t)$ is

(A) $(1-t)^{-1}$
(B) $(1-t)^{-2}$
(C) $(1-2t)^{-2}$
(D) $(1-|t|)$

20. A r.v. X has the p.d.f $f(x)=\frac{1}{x^2}$, $1<x<\infty$. Then $E(X)$ is

(A) 1
(B) 1/2
(C) 4
(D) Does not exist

21. Assume that for a r.v. X, $E(X-b)^2$ exists for all real b. Then $E(X-b)^2$ is minimum when b is equal to

(A) mean of X
(B) median of X
(C) mode of X
(D) 0

22. A r.v. X has the m.g.f. of $M(t)=(1-t)^{-2}$. Then $E(X)$ is

(A) 1
(B) 1/2
(C) 4
(D) 2

23. Let $M(t)$ be the m.g.f. of a distribution and let $\psi(t)=\ln M(t)$. Then $\psi'(0)$ is

(A) 1
(B) 0
(C) $E(X)$
(D) $-E(X)$

24. A r.v. X assumes values −1, 0, 1 with probabilities 1/8, 6/8 and 1/8 respectively. Then $Var(X)$ is

(A) 1/8
(B) 1/4
(C) 1/2
(D) 1/16

25. A r.v. X assumes values −1, 0, 1 with probabilities 1/8, 6/8 and 1/8 respectively. Then $P(|X|\geq 1)$ is

(A) 1/8
(B) 1/4
(C) 1/2
(D) 1

26. A and B are two events. The probability that exactly one of these two events will occur is

(A) $P(A)+P(B)$
(B) $P(A)+P(B)-P(A\cap B)$
(C) $P(A\cap B^C)+P(A^C\cap B)$
(D) $P(A\cap B)+P(A\cup B)$

27. From five cards numbered 1 to 50, two cards are drawn without replacement. The probability of getting the cards with numbers which are multiples of 4 is

(A) 6/25
(B) 13/50
(C) 124/2450
(D) 66/1225

28. Given $P(A)=0.5$, $P(B)=0.3$, A and B are independent. Then the probability that either A or B will occur is

(A) 0.15
(B) 0.95
(C) 0.65
(D) 0.80

29. With the usual notation if $P(|X-\mu|\geq c)\leq 1/k^2$, then c is

(A) ε
(B) $k\sigma$
(C) $k\sigma^2$
(D) $1/k\sigma$

30. The G.M. of three numbers 2/3, 3/10 and 81/25 is

(A) 9/5
(B) 3/5
(C) 3/25
(D) $\left(\frac{3}{5}\right)^{1/3}$

31. The harmonic mean of three numbers 3, 5, 9 is

(A) 405/17
(B) 135/17
(C) 17/135
(D) 17/405

32. In a lot of 1000 bulbs 200 are defective. A sample of 15 bulbs is selected from the lot. Then the expected number of defective bulbs in the sample is

(A) 1.5
(B) 30
(C) 20
(D) 10

33. Let the r.v. X follow $B(6,0.4)$. Then $Var(X)$ is

(A) 1.44 (B) 1.2
(C) 2.4 (D) 14.4

34. Let X follow $P(\lambda)$. Given $P(X=3)=0.2$ and $P(X=4)=0.4$. Then the value of λ is

(A) 2 (B) 4
(C) 16 (D) 8

35. The p.m.f of a r.v. Y is $p(y)=p(1-p)^y$, $y=0,1,2,....$. The distribution of Y is

(A) Bernoulli (B) negative binomial
(C) hyper geometric (D) uniform

36. The m.g.f. of a r.v. X is $M(t)=e^{4t+4t^2}$. Then the distribution of X is

(A) N(4, $\sqrt{8}$) (B) N(4, 4)
(C) N(4, 8) (D) N(4,8^2)

37. A r.v. X has the p.d.f $f(x)=\frac{xe^{-x/2}}{4}, x>0$. Then the distribution of X is

(A) Weibull (B) Beta
(C) $\chi^2(4)$ (D) $\chi^2(2)$

38. Let $X_1, X_2,....,X_n$ be a random sample from a distribution with m.g.f. $M(t)$. Then the m.g.f. of $\sum_{i=1}^{n} X_i/n$ is

(A) $\{M(t)\}^n$ (B) $\{M(t/n)\}^n$
(C) $\{M(t/n)\}$ (D) $M(t)/n$

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39. Let S_n^2 denote the variance of a random sample of size n from $N(\mu, \sigma^2)$. Then $\frac{nS_n^2}{n-1}$ converges stochastically to

(A) $\chi^2_{(n)}$
(B) $\chi^2_{(n-1)}$
(C) $t_{(n)}$
(D) σ^2

40. Let (X, Y) have a bivariate p.d.f given by $f(x, y) = 2,\ 0 < x < y < 1$. The conditional distribution of X given $Y = y$ on an appropriate range is

(A) 1
(B) $1/y^2$
(C) $1/(2-y)$
(D) $1/y$

41. A r.v. X has the p.d.f $f(x) = \frac{e^{-x/\beta}}{\beta}, x > 0$. Then $E(X)$ is

(A) 2β
(B) β
(C) $1/\beta^2$
(D) $1/\beta$

42. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with p.d.f. $f(x, \theta) = \theta e^{-\theta x},\ x > 0$. Then the m.l.e. of θ is

(A) $\bar{X}$
(B) $X_{(1)}$
(C) $1/\bar{X}$
(D) $\bar{X}e^{\bar{X}}$

43. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \theta)$. Then a sufficient statistic for θ is

(A) $\pi_{i=1}^{n} x_i$ (B) $\pi_{i=1}^{n} x_i / n$

(C) $\sum_{i=1}^{n} x_i$ (D) $\sum_{i=1}^{n} x_i^2$

44. The p.m.f. of a r.v. X is $p(x) = pq^x$, $x = 0, 1, 2, \ldots$. Then $E(X)$ is

(A) q/p (B) q/p^2

(C) $1/p^2$ (D) $1/p$

45. Let X_1 follow $B(n_1, p_1)$ and X_2 follow $B(n_2, p_2)$. Assume X_1 and X_2 are independent. Then the distribution of $X_1 + X_2$ is

(A) Bernoulli (B) Binomial

(C) Poisson (D) None of the above

46. Let X_1 follow Gamma(α_1, β_1) and X_2 follow Gamma(α_2, β_2). Then the distribution of $X_1 + X_2$ is

(A) Gamma$(\alpha_1 + \alpha_2,\ \beta_1 + \beta_2)$

(B) Gamma$(\alpha,\ \beta_1 + \beta_2)$ where $\alpha = \min(\alpha_1, \alpha_2)$

(C) Gamma$(\alpha_1 + \alpha_2,\ \beta)$ where $\beta = \min(\beta_1, \beta_2)$

(D) None of the above

47. The mean weight of 80 boys is 60 kg and mean weight of 70 girls is 72 kg. Then the mean weight of all the 150 persons is

(A) 65.6 kg (B) 64 kg

(C) 68.5 kg (D) 66.5 kg

48. For a given series mean is 4 and C.V. is 61.25%. Then the S.D. is

(A) 15.3125
(B) 24.5
(C) 2.45
(D) 1.5652

49. For a given data set the mean is 6, median is 5.8, mode is 5 and S.D. is 2. Then the coefficient of skewness is

(A) -0.5
(B) 0.5
(C) 0.4
(D) 0.1

50. The variance of four observations is 5.5. If all the observations are multiplied by 10, then the variance of the new set is

(A) 55
(B) 550
(C) 0.55
(D) 5.5

51. Let X follow $\text{Beta}_1(m,n)$. Then $E(X)$ is

(A) 2mn
(B) $\frac{mn}{m+n}$
(C) $\frac{m}{m+n}$
(D) $\frac{m+n}{m}$

52. Let X follow χ^2_m, Y follow χ^2_n, X and Y are independent. Then the distribution of X/Y is

(A) Gamma
(B) $\text{Beta}_1(m,n)$
(C) $\text{Beta}_2(m,n)$
(D) Chi-square

53. Expenditure during first five months of a year is Rs.96 per month and during the last seven months is Rs.120 per month. The average expenditure per month during whole year is

(A) Rs.110
(B) Rs.108
(C) Rs.100
(D) Rs.124

54. There were 25 teachers in a school whose mean age was 30 years. A teacher retired at the age of 60 years and a new teacher was appointed in his place. The mean age of teachers in the school was reduced by one year. The age of the new teacher was

(A) 25 years
(B) 30 years
(C) 35 years
(D) 32 years

55. With the usual notations if $Z = \dfrac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$, then Z is called the

(A) Laspeyre's price index number
(B) Laspeyre's quantity index number
(C) Paasche's price index number
(D) Paasche's quantity index number

56. For a 2^3 factorial experiment in a particular replicate in two blocks the treatments are arranged as follows. Which treatment is confounded in these blocks?

Block 1: (abc) (b) (ac) (1)
Block 2: (c) (ab) (a) (bc)

(A) (ac)
(B) (bc)
(C) (abc)
(D) (ab)

57. Let $X_1, X_2, ..., X_n$ be a random sample from the distribution with p.d.f. $f(x,\theta) = \frac{1}{\theta} e^{-(x-\mu)}$, $x > \mu$ and $H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$. Here

(A) both H_0 and H_1 are simple
(B) H_0 is simple and H_1 is composite
(C) H_0 is composite and H_1 is simple
(D) both H_0 and H_1 are composite

58. Two dice are thrown. What is the probability of getting a total as a multiple of 4 ?

((A) 1/6
(B) 1/2
(C) 2/3
(D) 4/36

59. In a city 50% read newspaper A, 30% read newspaper B and 20% read newspaper C, 20% read A and B, 30% read A and C, 10% read B and C. Also 15% read papers A, B and C. The percentage of people who do not read any of these papers is

(A) 30%
(B) 95%
(C) 55%
(D) 45%

60. In stratified random sampling under optimum allocation the stratum sample size n_h is proportional to

(A) N_h
(B) S_h
(C) $\sum_h N_h S_h$
(D) $N_h S_h$

61. Simple random sampling is advantageous when

(A) the sampling frame is not readily available
(B) the population under study is homogeneous
(C) the population under study is heterogeneous
(D) the population size is small

62. The p.d.f. of a r.v. is given by $f(x) = \frac{1}{2}\exp(-|x|),\ -\infty < x < \infty$. The distribution is called

(A) negative exponential
(B) Weibull
(C) logistic
(D) Laplace

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63. Let X follow $N(0,1)$. Then $Y = X^2$ has the following distribution

(A) Cauchy
(B) Chi-square
(C) Lognormal
(D) Laplace

64. A random variable X is Poisson with parameter λ. If $\hat{\lambda}$ is the m.l.e. of λ based on a sample of size n, then $Var(\hat{\lambda})$ is

(A) $1/n\lambda$
(B) λ^2/n
(C) $1/n\lambda^2$
(D) λ/n

65. Inversion formula is used to find

(A) the characteristic function (c.f.) given the d.f.
(B) the c.f. given the p.m.f. of a discrete distribution
(C) the d.f. from the c.f.
(D) the standard error of the estimator

66. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, $\sigma^2 > 0$. Let $Z_n = \sum_{i=1}^{n} X_i$. Then the limiting distribution of Z_n is

(A) $N(\mu, \sigma^2/n)$
(B) $t-$ distribution
(C) standard normal distribution
(D) Does not exist

67. In time series analysis ratio to trend method is used for the measurement of

(A) trend
(B) cyclical variation
(C) seasonal variation
(D) random variation

68. The np-chart is appropriate for

(A) number of defects
(B) variable sample size
(C) fraction defective
(D) None of the above

69. Let X_1 and X_2 be a random sample from a distribution having the p.d.f. $f(x)=e^{-x}$, $0<x<\infty$. Then the distribution of $W=\min(X_1, X_2)$ is

(A) exponential
(B) gamma
(C) Erlang
(D) beta

70. Let X follow $N(\mu,\sigma^2)$. For testing $H:\sigma^2=\sigma_o^2$, the most powerful test is based on the distribution

(A) normal
(B) t
(C) Chi-square
(D) F

71. The characteristic function of a random variable X is $\frac{1}{1+t^2}$. Then X has the following distribution:

(A) Laplace
(B) Cauchy
(C) Exponential
(D) Uniform

72. Let X follow $N(\mu,\sigma^2)$. Then $t=\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}$ is

(A) an unbiased estimator of σ^2
(B) a consistent estimator of σ^2
(C) least squares estimator of σ^2
(D) UMVUE of σ^2

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73. A r.v X follow $U(0,\theta)$. Then the m.l.e of θ is

(A) $X_{(n)}$
(B) $1/\overline{X}$
(C) $X_{(1)}$
(D) $\overline{X}$

74. If X is binomial with parameters n and $p=1/2$. If $P(X=4)=P(X=5)$, then

(A) $n=6$
(B) $n=8$
(C) $n=10$
(D) $n=9$

75. Let $F(x)$ be the d.f. of a continuous r.v. X. Then $F(X)$ follows

(A) Exponential
(B) Normal
(C) Uniform
(D) None of the above

76. A r.v. X has the p.d.f. $f(x)=1/3,\ 1\le x\le 4$. Then the median of X is

(A) 1/2
(B) 5/2
(C) 7/2
(D) 2

77. A cyclist pedals from his house to his college at a speed of 20 km.p.h. and back from the college to his house at 25 km.p.h. Then the average speed is

(A) 22.75 km.p.h.
(B) 22.5 km.p.h
(C) 22.22 km.p.h.
(D) 22.62 km.p.h.

78. John is three times as old as his son. After ten years, the sum of their ages will be 76 years. Then the present age of John is

(A) 42 years
(B) 38 years
(C) 52 years
(D) 56 years

79. If $\varphi_x(t)$ is a characteristic function, then $\varphi_x(0)$ is

(A) 0
(B) ∞
(C) 1
(D) $E(X)$

80. Let the joint p.m.f. of X and Y be $f(x,y)=(x+y)/21,\ x=1,2,3$ and $y=1,2$. Then $P(X=3)$ is

(A) 2/7
(B) 5/14
(C) 3/14
(D) 3/7

81. Given that the probability that a basket ball player throws the ball correctly and gets a score in any throw is 0.4. What is the probability that he gets the 2nd score exactly at the 4th throw?

(A) 0.1382
(B) 0.0576
(C) 0.1162
(D) 0.1728

82. Let X and Y be two related variables. The two regression lines are given by $x-y+1=0$ and $2x-y+4=0$. The two regression lines pass through the point

(A) (1,−2)
(B) (−3,2)
(C) (−3,−2)
(D) (2,3)

83. The series $\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+\frac{1}{7^p}+....$ is convergent for

(A) $p=0$
(B) $p=-1$
(C) $p=2$
(D) $p<1$

84. Identify the odd item in the following:

(A) Local control
(B) Replication
(C) Randomisation
(D) Confounding

85. The purpose of replication is to

(A) estimate the missing observations
(B) eliminate the interaction effect
(C) average out the influence of chance factors
(D) average out the effect of treatments

86. Among the following statements which statement is not true with respect to an m.l.e.?

(A) m.l.e. is consistent
(B) m.l.e. is unbiased
(C) m.l.e. satisfies invariance property
(D) m.l.e. is asymptotically normal

87. The probability distribution underlying the control limits for the C-chart is

(A) normal
(B) binomial
(C) Poisson
(D) Chi-square

88. Find the odd item in the following, related to control charts

(A) Control limits
(B) Warning limits
(C) Probability limits
(D) Specification limits

89. The intersection of two ogive curves determines the

(A) mean
(B) mode
(C) median
(D) C.D.

90. Among the following, which test is a non-parametric test?

(A) Chi-square test
(B) t-test
(C) Z-test
(D) F-test

91. The degrees of freedom for the t-test for the equality of two population means with 10 observations on X and 8 observations on Y is

(A) 15
(B) 16
(C) 17
(D) 18

92. The degrees of freedom for the error sum of squares in RBD with 4 treatments and 5 blocks is

(A) 15
(B) 12
(C) 14
(D) 16

93. If $P(A)=0.9$, $P(B)=0.8$, then $P(AB)$ is

(A) greater than 0.7
(B) greater than or equal to 0.7
(C) less than 0.7
(D) equal to 0.7

94. Let $P(A)=p_1$, $P(B)=p_2$, $P(A\cap B)=p_3$ with $p_1, p_2, p_3>0$. Then $P(A^c\cap B^c)$ is equal to

(A) p_1+p_2
(B) p_1-p_2
(C) $1-p_1-p_2-p_3$
(D) $1+p_1-p_2+p_3$

95. If A and B are independent events, which one of the statements is not true?

(A) A and B^c are independent
(B) A^c and B are independent
(C) A and $(A-B)$ are independent
(D) A^c and B^c are independent

96. A bag contains 3 red, 5 black and 7 yellow balls. If a ball is selected at random, then the probability that the ball drawn is not yellow is

(A) 7/15
(B) 8/15
(C) 7/8
(D) 1/7

97. If a two digit number 'k' is 4 times the sum of its digit and 2 times the product of its digit, then the number is

(A) 36 (B) 48
(C) 20 (D) 45

98. A cricket team consisting of 11 players is to be selected from 6 batsmen, 6 bowlers, 2 all rounders and 1 wicket keeper. 10 players, namely, 4 batsmen, 4 bowlers, 1 all rounder and 1 wicket keeper get selected in the team based on their previous performance. What is the probability that the remaining place is filled by a batsman?

(A) $\frac{1}{4}$ (B) $\frac{1}{5}$

(C) $\frac{1}{15}$ (D) $\frac{2}{5}$

99. A random variable X takes the values 1, 2, 3,... and $P(X=x)=1/2^n$; $x=1, 2, 3,...$ Then $P(X \text{ is divisible by } 5)$ is equal to

(A) 2/31 (B) 3/31
(C) 1/31 (D) 1/7

100. Given $P(A \cup B)=5/6$, $P(A \cap B)=1/3$, $P(B^c)=1/2$. Then the events A and B are

(A) mutually exclusive (B) A is a sub event of B
(C) independent (D) equally likely

101. The probability density function of X is $f(x) = \begin{cases} \frac{1}{4}, & |x| < 2 \\ 0, \text{otherwise} \end{cases}$.

Then $P(2X+3>5)$ is equal to

(A) 1/3 (B) 1/2
(C) 1/7 (D) 1/4

102. If $f(x)=1/\pi; 0 \le x \le \pi$, then $E(\text{Sin}x)$ is equal to

(A) $2/\pi$
(B) $3/\pi$
(C) $1/\pi$
(D) 0

103. If the Cov $(X,Y)=3$, then the covariance between $(2X+3)$ and $(4Y+2)$ is equal to

(A) 24
(B) 12
(C) 9
(D) 20

104. Let $f(x,y)=\begin{cases} 24xy; & x>0,\ y>0,\ x+y\le 1 \\ 0 & \text{otherwise} \end{cases}$. Then the conditional density of Y given $X=x$ is

(A) $\frac{2y}{(1-x)^2}\ ; 0<y<1-x$
(B) $\frac{2y}{(1-x)^2}\ ; 0<y<1+x$
(C) $\frac{(1-x)^2}{2y}\ ; 0<y<1$
(D) $\frac{(1-x)^2}{2y}\ ; 0<x<1$

105. The moment generating function (m.g.f.) of a binomial random variable is given by $(\frac{1+2e^t}{3})^5$. Then $p(X=2)$ is

(A) 40/243
(B) 42/243
(C) 30/243
(D) 29/243

106. If X and Y are independent binomial (5, 1/2) and binomial (7, 1/12), then $P(X+Y=3)$ is equal to

(A) $55/2^{11}$
(B) $55/2^{10}$
(C) $55/2^{12}$
(D) $55/2^{13}$

107. Let $X_1 \sim N(\mu = 2, \sigma^2 = 1)$ and $X_2 \sim N(\mu = 3, \sigma^2 = 2)$. Then the distribution of $2X_1 + 3X_2$ is

(A) $N(12, 15)$ (B) $N(15, 12)$
(C) $N(22, 13)$ (D) $N(13, 22)$

108. Let X follow uniform distribution over the interval (2,4). Then the mean and variance are

(A) 3/2, 3/12 (B) 2/3, 4
(C) 1/3, 6 (D) 1/3, 2

109. A pair of distributions satisfying memoryless property is

(A) exponential and gamma (B) geometric and Chi-square
(C) exponential and geometric (D) exponential and normal

110. The p.d.f. of a r.v. X is $f(x) = 2e^{-2x}$, $x > 0$. Then $F(2)$ is

(A) $\frac{e^4 - 1}{e^4}$ (B) $\frac{e - 1}{e}$

(C) $\frac{e^3 - 1}{e}$ (D) $\frac{e + 1}{e - 1}$

111. The arithmetic mean and geometric mean of two observations are 5 and 4 respectively. Then the observations are

(A) 2,8 (B) 4,1
(C) 6,4 (D) 3,7

112. The harmonic mean of 1, 1/2 , 1/3,…,1/n is

(A) n (B) $2n$
(C) $2/(n+1)$ (D) $n(n+1)/2$

113. If arithmetic mean and coefficient of variation of x are 20 and 20 respectively, then what is the variance of $y = 10 - 2x$?

(A) 64
(B) 16
(C) 36
(D) 84

114. If the range of a set of observations, x is 2, then the range of $-3x + 50$ is

(A) 2
(B) -6
(C) 44
(D) $+6$

115. If the values of 1^{st} and 3^{rd} quartiles are 20 and 30 respectively, then the value of inter quartile range is

(A) 10
(B) 25
(C) 5
(D) 0

116. The arithmetic means of x and y is 80 and 98 respectively and the variance of x and y is 4 and 9 respectively. If the value of the correlation coefficient between x and y is obtained as 0.6, then what is the most likely value of y when $x = 90$?

(A) 90
(B) 103
(C) 104
(D) 107

117. For a bivariate set of 5 observations, if the sum of squares of difference in ranks is obtained as 24, then the value of rank correlation coefficient is

(A) 0.2
(B) -0.4
(C) 0.40
(D) -0.2

118. If the regression lines of y on x and x on y are identical, then the correlation coefficient between x and y is

(A) $+1$
(B) -1
(C) ± 1
(D) 0

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119. If e_x is the expectation of life at age x, then which one of the following is true?

(A) $e_x + x = e_0$
(B) $e_x + x > e_0$
(C) $e_x + x < e_0$
(D) e_x is an increasing function

120. By suitably selecting the width, the moving averages of a time series can be made to be free from the effects of

(A) trend and seasonal variation
(B) seasonal and irregular variation
(C) trend only
(D) trend and irregular variation

121. Product control is achieved through

(A) control charts
(B) acceptance sampling plans
(C) a study of assignable causes of variation in quality
(D) a study of tolerance limits

122. Which index number (IN) satisfies the factor reversal and time reversal tests?

(A) Paasche's IN
(B) Laspeyres IN
(C) Marshall-Edgeworth IN
(D) Fisher's IN

123. Stratified random sampling is recommended where the population is

(A) homogeneous
(B) non-homogeneous
(C) non-homogeneous but can be divided into homogeneous sub-populations
(D) having a linear trend

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134. The variances of $\overline{x}_{st}$ under random sampling, proportional allocation and Neyman allocation are related as

(A) $V_{\text{ran}}(\overline{x}_{st}) < V_{\text{prop}}(\overline{x}_{st}) < V_{\text{Ney}}(\overline{x}_{st})$
(B) $V_{\text{ran}}(\overline{x}_{st}) = V_{\text{prop}}(\overline{x}_{st}) = V_{\text{Ney}}(\overline{x}_{st})$
(C) $V_{\text{ran}}(\overline{x}_{st}) > V_{\text{prop}}(\overline{x}_{st}) < V_{\text{Ney}}(\overline{x}_{st})$
(D) $V_{\text{ran}}(\overline{x}_{st}) \geq V_{\text{prop}}(\overline{x}_{st}) \geq V_{\text{Ney}}(\overline{x}_{st})$

135. A control chart for the number of defects is

(A) c-chart
(B) p-chart
(C) np-chart
(D) R-chart

136. If $P(\text{reject a lot} | p_0) = \alpha$ and $P(\text{accept a lot} | p_1) = \beta$, for the SPRT for testing $H_0 : p = p_0$ *vs* $H_1 : p = p_1$, the OC function is

(A) $L(p_0) = 1 - \beta$
(B) $L(p_0) = \alpha$
(C) $L(p_0) = \beta$
(D) $L(p_0) = 1 - \alpha$

137. Let ${}_nD_x$ be the number of deaths in the age group $(x, x+n)$ and ${}_nP_x$ is the total population of the age group x to $x+n$. Then the age specific death rate for the age group x to $x+n\left({}_nm_x\right)$ is given by

(A) $\frac{{}_nD_x}{{}_nP_x} \times 1000$
(B) $\frac{{}_nP_x}{{}_nD_x} \times 1000$
(C) $\frac{{}_nP_x}{{}_nD_x} \times 100$
(D) $\frac{{}_nD_x}{{}_nP_x} \times 100$

138. The first moment about the value 1.5 of the variable in a frequency distribution is 4.5. The mean is

(A) 5.5
(B) 6
(C) 6.5
(D) 3

124. In the case of stratified random sampling under Neyman's optimum allocation, more sample observations are drawn from a stratum if

(A) the stratum size is large
(B) the stratum variability is large
(C) Both (A) and (B) are true
(D) None of the above

125. To compare several treatments, when the experimental units are homogeneous, the appropriate design to be used is

(A) Completely Randomised Design
(B) Randomised Block Design
(C) Latin Square Design
(D) Split Plot Design

126. In any statistically designed experiment replication of treatments is necessary because then only

(A) experimental error can be estimated
(B) the variation due to treatment effects can be estimated.
(C) randomisation and local control can be effectively incorporated
(D) None of the above is true

127. The concept of sufficiency in Statistical Inference was introduced by

(A) Ronald Fisher
(B) Karl Pearson
(C) Jersy Neyman
(D) Mahalanobis

128. For an estimator T_n of θ to be consistent, the conditions $E(T_n) \to \theta$ and $Var(T_n) \to 0$ as $n \to \infty$ are

(A) necessary conditions
(B) sufficient conditions
(C) necessary and sufficient conditions
(D) neither necessary nor sufficient

119. If e_x is the expectation of life at age x, then which one of the following is true?

(A) $e_x + x = e_0$
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123. Stratified random sampling is recommended where the population is

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(C) non-homogeneous but can be divided into homogeneous sub-populations
(D) having a linear trend

129. If T_n is unbiased and consistent for θ, then

(A) T_n^2 is unbiased and consistent for θ^2
(B) T_n^2 is unbiased but not consistent for θ^2
(C) T_n^2 is biased but consistent for θ^2
(D) T_n^2 is biased and not consistent for θ^2

130. A hypothesis is rejected at the level of significance $\alpha = 5\%$ by a test. Then which one of the following statements is true regarding the p-value of the test?

(A) $p = 5\%$
(B) $p < 5\%$
(C) $p > 5\%$
(D) Any one of the above three can be true

131. Which one of the following test is used to test whether an observed correlation coefficient is significantly different from zero?

(A) A test based on standard normal distribution
(B) A test based on Chi-square distribution
(C) A test based on t-distribution
(D) A test based on F-distribution

132. The power of a test will depend on

(A) the hypothesis tested
(B) the alternate hypothesis
(C) both the alternate hypothesis and the hypothesis tested
(D) the level of significance specified

133. T is the minimum variance bound estimator of θ. That is Var (T) = Cramer Rao bound for the variance of unbiased estimators of θ. Which one of the following statements closely summarises the properties of T?

(A) T is unbiased for θ
(B) T is sufficient for θ
(C) T is consistent for θ
(D) All of the above

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(D) $\frac{{}_nD_x}{{}_nP_x} \times 100$

138. The first moment about the value 1.5 of the variable in a frequency distribution is 4.5. The mean is

(A) 5.5
(B) 6
(C) 6.5
(D) 3

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139. The mean and standard deviation of a Chi-square distribution with 8 degrees of freedom are respectively

(A) 8,16
(B) 8, 4
(C) 4, 4
(D) 4, 8

140. The producer's risk is the probability of

(A) rejecting a good lot
(B) accepting a good lot
(C) rejecting a bad lot
(D) accepting a bad lot

141. If T_1 is an MVUE of $\gamma(\theta); \theta \in \Theta$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency e_θ, the correlation coefficient between T_1 & T_2 is equal to

(A) e_θ
(B) $e_\theta^{\,2}$
(C) $\dfrac{1}{\sqrt{e_\theta}}$
(D) $\sqrt{e_\theta}$

142. A valid t-test to assess an observed difference between two sample mean values requires that
(i) both populations are independent.
(ii) the observations to be sampled from normally distributed parent population.
(iii) the variance to be the same for both populations.

(A) (i) and (ii)
(B) (ii) and (iii)
(C) (i) and (iii)
(D) all the three conditions

143. If X follows an F distribution with (2, 4) degrees of freedom, then $\dfrac{1}{X}$ follows

(A) an F distribution with (4, 2) degrees of freedom
(B) an F distribution with (2, 4) degrees of freedom
(C) a student's t-distribution with 6 degrees of freedom
(D) a Chi-square distribution with 2 degrees of freedom

144. Let $X_1, X_2, \ldots, X_n$ be a random sample from $B\,(1,\,p)$. Then the consistent estimator of $p(1-p)$ is

(A) $\overline{X}$
(B) $\overline{X}^2$
(C) $\overline{X}(1-\overline{X})$
(D) $n.\overline{X}$

145. Attributes A and B are said to be positively correlated if

(A) $\frac{(AB)}{(B)} < \frac{(A\beta)}{(\beta)}$
(B) $\frac{(AB)}{(B)} > \frac{(A\beta)}{(\beta)}$
(C) $\frac{(AB)}{(A)} < \frac{(A\beta)}{(\beta)}$
(D) $\frac{(AB)}{(A)} > \frac{(A\beta)}{(\beta)}$

146. The variance of the first n natural numbers is

(A) $\frac{n^2-1}{12}$
(B) $\frac{n^2+1}{12}$
(C) $\frac{n(n+1)(2n+1)}{6}$
(D) $\left\{\frac{n(n+1)}{2}\right\}^2$

147. A sampling design which ensures administrative convenience and fixed sample size is

(A) linear systematic sampling
(B) circular systematic sampling
(C) stratified random sampling
(D) cluster sampling

148. For a bivariate non-negative random vector (X,Y) with distribution function $F(x,y)$, denote by $\overline{F}(x,y)=P(X>y, Y>y)$. Then $\overline{F}(x,y)$ is equal to

(A) $1-F(x,y)$

(B) $1-F(x,o)-F(o,y)$

(C) $1-F(x,o)-F(o,y)+F(x,y)$

(D) $1+F(x,o)+F(o,y)-F(x,y)$

149. The founder Chairman of the Planning Commission of India was

(A) Pandit Jawaharlal Nehru
(B) Sardar Vallabhai Patel
(C) Smt. Indira Gandhi
(D) Sri P.C. Mehalanobis

150. Choose the wrong statement from among those given below.

(A) The characteristic function always exists
(B) The exponential distribution is a special case of the Gamma distribution
(C) The student's t distribution is positively skewed
(D) For a bivariate normal distribution, non-correlation implies independence and vice versa

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