61213

ROLL No. | | | | | |

TEST BOOKLET No. 596

TEST FOR POST GRADUATE PROGRAMMES

MATHEMATICS

Time: 2 Hours | Maximum Marks: 450

INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.
2. Write your Roll Number in the space provided on the top of this page.
3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the Answer Sheet. Darken the appropriate bubbles with a **Ball Point Pen.**
4. The paper consists of 150 objective type questions. All questions carry equal marks.
5. Each question has four alternative responses marked A, B, C and D and you have to darken the bubble fully by a **Ball Point Pen** corresponding to the correct response as indicated in the example shown on the Answer Sheet.
6. Each correct answer carries 3 marks and each wrong answer carries 1 minus mark.
7. Space for rough work is provided at the end of this Test Booklet.
8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.
9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of any such unforeseen happenings, the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

MATHEMATICS

1. The linear function $f(x) = mx + b$ where $m \neq 0$ is

(A) continuous everywhere (B) not continuous
(C) continuous when $m > 0$ (D) None of the above

2. The absolute extreme values of $f(x) = x^3 - 3x + 2$ on [0,2] are

(A) 0, 4 (B) −1, 4
(C) 4, 2 (D) 0, 3

3. The point on the parabola $y = (x-3)^2$ that is nearest the origin

(A) (2,−1) (B) (2, 1)
(C) (−2, 1) (D) (−2, −1)

4. $\lim_{x \to \infty} x^3 \sin\left(\frac{1}{2}\right)$

(A) 1 (B) $-\infty$
(C) ∞ (D) None of the above

5. The last digit of 2007^{2008} is

(A) 9 (B) 7
(C) 3 (D) 1

6. When an article is sold at b rupees and a paise after purchasing it at a rupees and b paise, the loss incurred is c rupees. Then the correct relation among a, b, c is

(A) $\frac{a+b}{c} = 0.99$ (B) $\frac{c}{a+b} = 0.99$
(C) $\frac{c}{a-b} = 0.99$ (D) $\frac{a-b}{c} = 0.99$

7. 852 digits are used to number the pages of a book consecutively from page 1. The number of pages in the book is

(A) 284 (B) 316
(C) 320 (D) 351

8. The number of zeros used in writing natural numbers from 1 to 200 is

(A) 21 (B) 25
(C) 31 (D) 41

9. There will be a CAT-2012 (Common Admission Test) in the University. Let CAT consist of three distinct positive integers C, A, T such that the product $C \times A \times T = 2012$. The largest possible value of the sum C + A + T is

(A) 508 (B) 1006
(C) 1009 (D) 2014

10. Bhaskar distributes some chocolates to four boys in the ratio 1/3 1/4 1/5 1/6. The minimum number of chocolates that Bhaskar should have is

(A) 23 (B) 46
(C) 57 (D) 60

11. In the following display | 2 | B | C | D | E | 6 |, each letter represents a digit. If the sum of any three successive digits is 15, then the value of E is

(A) 0 (B) 2
(C) 7 (D) 8

12. A rectangular sheet of metal, x cm by y cm, has a square of side z cm cut from each corner. The sheet is then bent to form a tray of depth z cm. The volume (in cc) of the tray is

(A) $z=(x-y)(y-z)$ (B) xyz
(C) $(x+y)z$ (D) $z(x-2z)(y-2z)$

13. If a circle and a square have the same perimeter, then the area of the circle is

(A) smaller
(B) π times the area of the square
(C) equal to the square
(D) larger

14. If a circle has radius 2 cm and a square is inscribed in the circle, then the area of the square (in sq. cm) is

(A) 2 (B) $2\sqrt{2}$
(C) 4 (D) 8

15. If $3^p = 4$, $4^q = 5$, $5^r = 6$, $6^s = 7$, $7^t = 8$ and $8^u = 9$, then the value of the product '$p\,q\,r\,s\,t\,u$' is

(A) 9 (B) 8
(C) 2 (D) 1

16. If A is the set of natural numbers and B is the set of integers, the symmetric difference of A and B is

(A) set of natural numbers (B) set of integers
(C) $\{0,-1,-2,\ldots\}$ (D) $\{-1,-2,-3,\ldots\}$

17. If A and B have 50 elements each and $A - B$ has 40 elements, then $n(A \cup B)$ is

(A) 80 (B) 90
(C) 95 (D) 100

18. 19 boys turn out for baseball. Of these 11 are wearing base ball shirts and 14 are wearing baseball pants. There are no boys without one or other. The number of boys wearing full uniform is

(A) 3 (B) 5
(C) 6 (D) 8

19. If X and Y are two sets, then $X \cap (Y \cup X)^C$ equals

(A) $X \cup Y$ (B) Y
(C) X (D) ϕ

20. Sets A and B have 3 and 6 elements each. The minimum number of elements in A∪ B can be

(A) 3 (B) 6
(C) 9 (D) 18

21. The value of $\begin{vmatrix} p & 5 & q+r \\ q & 5 & r+p \\ r & 5 & p+q \end{vmatrix}$ is

(A) $5+p+q+r$ (B) $5(p+q+r)$
(C) $p+q+r$ (D) 0

22. If $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$, then x is equal to

(A) $-\frac{2}{5}$ (B) $-\frac{5}{2}$

(C) $\frac{2}{5}$ (D) $\frac{5}{2}$

23. The value of the determinant $\begin{vmatrix} 51 & 52 & 53 \\ 52 & 53 & 54 \\ 53 & 54 & 55 \end{vmatrix}$ is

(A) 5 (B) 3

(C) 0 (D) −3

24. The integer co-ordinates of point whose distances from (4,6) and (6,−1) are respectively 5 and $\sqrt{34}$ are

(A) (3,2) (B) (2,5)

(C) (1, 2) (D) $(\sqrt{2}, \sqrt{3})$

25. The circle $x^2 + y^2 = 16$ meets the co-ordinate axes at A, B, C and D respectively. Then the area (in sq. unit) of the square $ABCD$ is

(A) 8 (B) 16

(C) 32 (D) 64

26. An equilateral triangle of side 9cm is inscribed in a circle. The radius of the circle is

(A) $\frac{9\sqrt{3}}{2}$ cm (B) $3\sqrt{3}$ cm
(C) $\frac{9}{4}\sqrt{3}$ cm (D) $9\sqrt{3}$ cm

27. The diagonal of a square A is $(x+y)$. The diagonal of square B with twice the area of A is

(A) $2(x+y)$ (B) $\sqrt{2}(x+y)$
(C) $2x+4y$ (D) $4x+2y$

28. If the length of common chord of two intersecting circles is 16 cm and their radii are 10 cm and 17 cm, then the distance between the centres is

(A) 715 cm (B) 21 cm
(C) $\sqrt{389}$ cm (D) 27 cm

29. The area and circumference of a circle are numerically equal. The radius of the circle is

(A) 22 unit (B) $\frac{22}{7}$ unit
(C) 3 unit (D) 2 unit

30. Suppose the area of the region bounded above by $y = f(x)$ and below by x-axis between the lines $x = 0$ and $x = 1$ equals A square units. The area bounded by $y = f(2x)$ on $\left[0, \frac{1}{2}\right]$ is

(A) A/4 (B) A/2
(C) A/6 (D) A

31. The centroid of the parabola region bounded by $y = ax^2$ and $y = b$ where $a > 0$ and $b > 0$ is

(A) independent of a
(B) dependent on a
(C) independent of a when $a > \left(\frac{2}{3}\right)$ and dependent on a when $a < \left(\frac{2}{3}\right)$
(D) None of the above

32. The set $\{1, sinx, cosx, sin2x, cos2x, \ ...\}$ is orthogonal on

(A) $\left[\frac{\pi}{2}, \frac{\pi}{4}\right]$
(B) $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$
(C) $[-\pi, \pi]$
(D) $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

33. $\int_0^{\infty} \frac{dx}{x^2+x}$

(A) converges to 2π
(B) converges
(C) diverges
(D) None of the above

34. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges for

(A) $p > 1$
(B) $p > \left(\frac{1}{2}\right)$ and $p > 1$
(C) $p \geq 1$
(D) None of the above

35. Let $\sum a_n$ be conditionally convergent.
Define $p_n = \begin{cases} a_n & \text{if } a_n \geq 0 \\ 0 & \text{if } a_n < 0 \end{cases}$ and $q_n = \begin{cases} -a_n & \text{if } a_n < 0 \\ 0 & \text{if } a_n \geq 0 \end{cases}$.
Then

(A) $\sum p_n$ converges
(B) $\sum q_n$ converges
(C) Both diverges
(D) Both converges

36. Interval of convergence of $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{(n+1)^2 3^n}$ is

(A) $[-1, 1]$ (B) $[-1, 2]\cup[3, 5]$
(C) $[-1, 5]$ (D) None of the above

37. Curvature and centre of curvature at $p(2,1)$ on the curve $r(t) = \langle 2t^2, 2-t^3\rangle$ where $t > 0$

(A) $\frac{2}{25}, \left(\frac{-7}{4}, \frac{-22}{3}\right)$ (B) $\frac{25}{12}, \left(\frac{4}{-17}, \frac{-22}{3}\right)$
(C) $\frac{25}{12}, \left(\frac{17}{4}, \frac{22}{3}\right)$ (D) None of the above

38. $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(A) elliptic hyperboloid of two sheets
(B) hyperboloid of two sheets
(C) elliptic cone
(D) None of the above

39. $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$

(A) exists (B) does not exist
(C) exists when $xy = 1$ (D) None of the above

40. If $z = \frac{xy}{x+y}$, $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$

(A) z (B) $\frac{z}{x}$
(C) $\frac{z}{y}$ (D) None of the above

41. $\int_0^1 \int_x^1 \sin y^2 dydx$

(A) $\int_0^1 \int_0^y \sin y^2 dxdy$ (B) $\int_0^1 \int_0^{y^2} \sin x^2 dxdy$

(C) $\int_0^1 \int_0^y \sin y \ \ dydx$ (D) None of the above

42. Integral $\iiint_Q 2xzdV$ where Q is the region enclosed by the planes $x + y + z = 4, y = 3x, x = 0$ and $z = 0$

(A) $\frac{1}{15}$ (B) $\frac{-16}{5}$

(C) $\frac{6}{15}$ (D) $\frac{16}{15}$

43. The value of $\iint_R (x + y)e^{x^2-y^2} dA$ where R bounded by $x + y = 1$, $x + y = 2$, $x^2 - y^2 = 1$ and $x^2 - y^2 = -1$ is

(A) $\cosh(1)$ (B) $\sinh(1)$

(C) $\tan(2)$ (D) $\sin(1)$

44. Value of $\int_C (e^x - x^2y)dx + \ (xy^2 + y^3)d,$ where C is the boundary of the semi annular region bounded by $y = \sqrt{1 - x^2}$ and $y = \sqrt{9 - x^2}$ oriented in a counter clockwise direction, is

(A) 2π (B) $20\pi^2$

(C) 20π (D) π

45. Flux of a fluid that has velocity field $F = 4xi + 4yj + 3zk$ through the parabolic surface S defined by $z = 4 - x^2 - y^2$ for $z \geq 0$ in the direction of outer unit normal is

(A) 80π (B) 8π

(C) 88π (D) π

46. $(2x - 3y)dx + ydy = 0,\ y(1) = 3$. Then

(A) $y = x + 2(y - 2x)^2$ (B) $y^2 = x + 2(y - x)^2$
(C) $y = x^2 + 2(y - x)^2$ (D) None of the above

47. Family of orthogonal trajectories of family of curves $3x^2 + y^2 = cx$ is

(A) $x^2 = y^2(1 - c'y)$ (B) $y^2 = x^2(1 - c'y)$
(C) $x^2 = y^2(1 - c'y^2)$ (D) None of the above

48. Particular solution of $y'' + 2y' - 8y = 5e^{3x}$ is $y_p = Ae^{ax}$

(A) A= 2/7 $a = 3$ (B) A= 5/7 $a = 3$
(C) A= 1/7 $a = 1$ (D) None of the above

49. Direction ratios of the line passing through the origin and the point $p(2,1,-3)$

(A) 2, 1, −3 (B) $\frac{2}{\sqrt{14}}\ \frac{1}{\sqrt{14}}\ \frac{-3}{\sqrt{14}}$
(C) 3, 3 , −1 (D) None of the above

50. $g.c.d[(5n + 3),\ 7n + 4]$, for all n

(A) $n + 3$ (B) Relatively prime
(C) $5n + 3$ (D) $7n + 4$

51. If a, b and c belong to group G and $ab = ca$, then $b = c$. Then

(A) G is non-abelian (B) G is cyclic
(C) G is abelian (D) None of the above

52. Set of all rational numbers of the form 3^m6^n where m and n are integers is a group under

(A) addition (B) multiplication
(C) division (D) None of the above

53. In any group, an element and its inverse have same order.

(A) True
(B) False
(C) For some elements in the group
(D) None of the above

54. In symmetric group S_3 let $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$. Then

(A) $\alpha\beta \neq \beta\alpha$
(B) $\alpha\beta = \beta$
(C) $\alpha^3 = \beta$
(D) None of the above

55. The mapping from R under addition to itself given by $\emptyset(x) = x^3$ $\emptyset$ is from group G onto a group $\overline{G}$

(A) not an isomorphism
(B) not homomorphism
(C) homomorphism
(D) A and B are true

56. Let $G = \{a + b\sqrt{2}: a, b \text{ rational}\}$ and $H = \left\{\begin{bmatrix} a & 2b \\ b & a \end{bmatrix}: a, b \text{ rational}\right\}$. G and H are isomorphic under

(A) subtraction
(B) multiplication
(C) addition
(D) None of the above

57 A sum of money amounts to Rs.6690 after 3yrs and to Rs.10035 after 6yrs on compound interest. The sum is

(A) Rs. 4459
(B) Rs. 4438
(C) Rs. 4460
(D) Rs. 4421

58. A car moves at the speed of 80Km/hr. The speed of car in meters per second is

(A) $22\frac{2}{9}$
(B) $\frac{22}{9}$
(C) $\frac{23}{2}$
(D) $\frac{11}{2}$

59. A walks at 4Kmph and 4 hrs after his start, B cycles after him at 10Kmph. How far from the start does B catch up with A?

(A) 27.6 Km (B) 26.7Km
(C) 23.5Km (D) 23Km

60. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36min, then slower pipe alone will be able fill the tank in

(A) 451min (B) 212min
(C) 144min (D) 145min

61. One man, 3 women and 4 boys can do a piece of work in 96 hrs, 2 men and 8 boys can do it in 80 hrs, 2 men and 3 woman can do it in 120hrs. 5 men and 12 boys can do it in

(A) 480/11hrs (B) 452/11 hrs
(C) 453/12 hrs (D) 23/21 hrs

62. 5 men and 2 boys working together can do four times as much work as a man and a boy. Working capacities of a man and a boy are in the ratio

(A) 3:1 (B) 2:1
(C) 2:2 (D) 3:4

63. In a diary farm, 40 cows eat 40 bags of husk in 40 days. One cow will eat one bag of husk in

(A) 13 days (B) 34 days
(C) 40 days (D) 32 days

64. If 3/5 of a cistern is filled in 1 minute, how much more time will be required to fill the rest of it?

(A) 12sec (B) 34sec
(C) 56sec (D) 40sec

65. Percentage of numbers from 1 to 70 that have squares that end in the digit 1

(A) 23 (B) 32
(C) 20 (D) 67

66. Ten years ago, A was half of B in age. If the ratio of their present ages is 3:4, total of their present ages is

(A) 12 yrs (B) 32 yrs
(C) 21 yrs (D) None of the above

67. The sum of the squares of two numbers is 3341 and the difference of their squares is 891. The numbers are

(A) 35, 46 (B) 30 , 23
(C) 34, 21 (D) 31, 23

68. How many iron rods, each of length 7m and diameter 2 cm can be made out of 0.88 cubicmetre of iron?

(A) 400 (B) 320
(C) 342 (D) 213

69. Of the four numbers whose average is 60, the first is one-fourth of the sum of the last three. The first number is

(A) 48 (B) 43
(C) 41 (D) 21

70. Two cones have their heights in the ratio 1:3 and radii 3:1. The ratio of their volumes is

(A) 1:2 (B) 3:1
(C) 2:3 (D) 3:4

71. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women?

(A) 12340 (B) 43214
(C) 11760 (D) 12334

72. The orthogonal trajectory of the family of circles $x^2+y^2=2cx$ (c is arbitrary) is described by the equation

(A) $(x^2+y^2)y'=2xy$ (B) $(x^2-y^2)y'=2xy$
(C) $(-x^2+y^2)y'=xy$ (D) $(-x^2+y^2)y'=2xy$

73. The distance moved by a particle is $s=t^3-6t^2-18t+12$. Then the velocity of the particle when acceleration is zero is

(A) −30 (B) 20
(C) 0 (D) −40

74. The radius of curvature of the curve $xy=c^2$ is

(A) $\frac{(x^2-y^2)^{\frac{3}{2}}}{2c^2}$ (B) $\frac{(x^2-y^2)^{\frac{2}{3}}}{2c^2}$
(C) $\frac{(x^2+y^2)^{\frac{3}{2}}}{2c^2}$ (D) $\frac{(x^2+y^2)^{2}}{2c^2}$

75. If y_1, y_2, y_3 be the ordinates of the vertices of a triangle inscribed in a parabola $y^2=4ax$, then the area of the triangle is

(A) $\frac{1}{8a}\prod(y_1-y_2)$ (B) $\frac{1}{16a}\prod(y_1-y_2)$
(C) $\frac{1}{32a}\prod(y_1-y_2)$ (D) $\frac{3}{16a}\prod(y_1-y_2)$

76. The value of the series $\sum_{n=1}^{\infty} \frac{1}{2n(2n-1)}$ is

(A) 1
(B) $1-\ln(2)$
(C) $\ln(2)-1$
(D) $\ln(2)$

77. If a and b are two relatively prime numbers, then $\phi(ab)$ is

(A) $\phi(a)\phi(b)$
(B) 1
(C) ab
(D) $\phi(a/b)$

78. The point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ is

(A) $(-1, 0, 0)$
(B) $(1, 1, 1)$
(C) $(-1, 1, 1)$
(D) $(-1, -1, 1)$

79. The function $f(x,y) = xy + 2x - ln(x^2 y)$ has the point $\left(\frac{1}{2}, 2\right)$ corresponds to, for $x > 0$ and $y > 0$

(A) local minimum
(B) local maximum
(C) global minimum
(D) global maximum

80. If $x = r\,cos0$ and $y = r sin\theta$, then $\frac{\partial(x,y)}{\partial(r,0)}$ is given by

(A) r
(B) $-r$
(C) $\frac{1}{r}$
(D) 1

81. $\int_0^1 (1-x)^{m-1} x^{n-1}\, dx$ is

(A) divergent
(B) convergent for all the values of m and n
(C) converges for $m > 0$ and $n > 0$
(D) converges for $m < 0$ and $n < 0$

82. $\frac{1}{2}\int_C (x\, dy - y\, dx)$ gives the

(A) volume enclosed by the curve
(B) area enclosed by the curve
(C) length of the curve
(D) surface area of the curve

83. The Laplace transform of the function $\frac{e^{-t}\sin t}{t}$ is

(A) $\tan^{-1} s$
(B) $\cot^{-1} s$
(C) $\tan^{-1}(s+1)$
(D) $\cot^{-1}(s+1)$

84. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, where ω is the cube root of unity, is

(A) 0
(B) 1
(C) ω
(D) $\frac{1}{\omega}$

85. The Taylor coefficients at the origin of the function $f(z) = \frac{1}{1-e^z}$ are

(A) rational numbers
(B) irrational numbers
(C) imaginary
(D) Taylor series does not exist

86. Let $A = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix} \in SL(2, Z_5)$. Then A^{-1} is

(A) $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

87. The set $\{1, 2, \ldots, n-1\}$ is a group under multiplication *modulo n*,

(A) if and only if n is odd (B) if and only if n is even
(C) if and only if n is prime (D) for all values of n

88. Let G be the group of nonzero real numbers under multiplication, $H = \{x \in G : x = 1 \text{ or } x \text{ is irrational}\}$ and $K = \{x \in G : x \geq 1\}$. Then

(A) both H and K are subgroups of G
(B) both H and K are not subgroups of G
(C) H is a subgroup of G but not K
(D) K is a subgroup of G but not H

89. Suppose a group contains elements a and b such that $|a| = 4$, $|b| = 2$ and $a^3 b = ba$. Then $|ab|$ is

(A) 2 (B) 4
(C) 3 (D) 5

90. Let a be a fixed element of a group G. Then the set of all elements in G that commute with a is

(A) normal subgroup of G (B) subgroup of G
(C) not a subgroup of G (D) None of the above

91 The order of the permutation $(1\ 2\ 4)(3\ 5\ 7\ 8)$ is

(A) 12 (B) 4
(C) 14 (D) 8

92. The principal value of $log(-1)$ is

(A) $\frac{i\pi}{2}$ (B) $\frac{i\pi}{3}$
(C) $i2\pi$ (D) $i\pi$

93. The principal value of $arg(1-i)$ is

(A) $\frac{3\pi}{2}$ (B) 2π
(C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{6}$

94. Radius of convergence of $\sum_{n=0}^{\infty}\frac{z^n}{n^n}$ is

(A) 1 (B) 0
(C) ∞ (D) $\frac{3}{4}$

95. All entire functions of the form $f(x,y) = u(x) + iv(y)$ is

(A) $(x+c_1) + i(y+c_2)$ (B) $(x+c_1)^2 + i(y+c_2)^2$
(C) $(x+c_1)^3 + i(y+c_2)^3$ (D) None of the above

96. The harmonic conjugate of a harmonic function $u(x,y) = x^2 - y^2$ is

(A) $x+y+c$ (B) $2xy+c$
(C) $(xy)^2+c$ (D) $\frac{x}{-}+c$

97. Suppose f is analytic in a region D such that at every point of D either $f = 0$ or $f' = 0$ Then

(A) f is constant
(B) f is non-constant
(C) f is not continuous
(D) None of the above

98. $\int_{c_1} \bar{z} dz$, where c_1 is a semicircular path from -1 to 1 is

(A) $\frac{i\pi}{2}$
(B) $\frac{\pi}{2}$
(C) $i\pi$
(D) π

99. $\int_{C} \tan z dz$, where the contour C is the circle $|z| = 1$, in either direction is

(A) π
(B) 0
(C) $\frac{\pi}{2}$
(D) $2\pi i$

100. $\int_C \frac{5z-2}{z(z-1)} dz$, when C is the circle $|z| = 2$ described counterclockwise, equals

(A) $2\pi i$
(B) 0
(C) ∞
(D) $10\pi i$

101. $\lim_{z \to \infty} \frac{2z+i}{z+1}$ is

(A) ∞
(B) i
(C) 2
(D) Does not exist

102. One of the cube root of $-8i$ is

(A) $\sqrt{3} \quad i$
(B) $4i$
(C) $\sqrt{3} + i$
(D) None of the above

103. Which of the following is true for the function $f(z) = |z|^2$?

(A) $f(z)$ is entire

(B) $f'(z)$ can't exist if $z \neq 0$

(C) Cauchy-Rieman equation is satisfied everywhere

(D) $f'(z)$ exists in the upper half plane

104. The arbitrary element for the set
$\{(a,b,c): a+b-c=0 \text{ and } 3a-b+5c=0\}$ is

(A) (−4t, 2t, t) for some t∈R (B) (−t, 2t, t) for some t∈R

(C) (t, t, 1) for some t∈R (D) None of the above

105. The set $U = \{a(2,-5) + b(3,-7): a, b \in R\}$ is

(A) the trivial subspace of V_2 (B) the proper subspace of V_2

(C) not a subspace of V_2 (D) None of the above

106. Suppose S and T are subspaces of a vector space V. Which of the following is a subspace of V?

(A) $S+T=\{U+V : U \in S, V \in T\}$

(B) $S \cup T=\{U : U \in S \text{ or } V \in T\}$

(C) $S.T=\{U.V : U \in S, V \in T\}$

(D) $S/T=\{U/V : U \in S, V \in T\}$

107. The span of $\{(2,-1,5),(0,3,-1)\}$ contains the vector

(A) $(2,5,0)$ (B) $(2,5,-1)$

(C) $(3,6,5)$ (D) None of the above

108. If S is a subset of a vector space V, then the span of S is

(A) empty (B) subspace of V
(C) need not be subspace of V (D) None of the above

109. The set $\{(2,5),(-1,1),(-4,-3)\}$ is

(A) linearly independent (B) linearly dependent
(C) proper subspace of V_2 (D) None of the above

110. Under what conditions on a and b, the set $\{(1,a,2),(1,1,b)\}$ linearly independent in V_3?

(A) $a=1,\ b=1$ (B) $a\neq 1,\ b\neq 1$
(C) $a\neq 1,\ b=2$ (D) $a\neq 1,\ b\neq 2$

111. The set $\{\ 1+t+t^2, 1+t\ , 1\}$ is

(A) a basis for $R_3[t]$
(B) not linearly independent in $R_3[t]$
(C) not a basis for $R_3[t]$
(D) None of the above

112. Which of the following is a linear transformation?

(A) T: $V_2 \to V_2$; $T(a,b) = a/b$
(B) T: $V_2 \to V_2$; $T(a,b) = ab$
(C) T:$V_2 \to V_2$; $T(a,b) = a^2+b^2$
(D) T: $V_2 \to V_2$; $T(a,b) = a+b$

113. Let z be a complex number satisfying $z^2 + z + 1 = 0$. If n is not a multiple of 3, then the value of $z^n + z^{2n}$ is

(A) -1 (B) 0
(C) -2 (D) 1

114. $a + ib > c + id$ is defined only when

(A) $a = 0$ and $c = 0$ (B) $c = 0$ and $d = 0$
(C) $a = 0$ and $d = 0$ (D) $b = 0$ and $d = 0$

115. The point of intersection of the lines given by
$x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ is

(A) $(1,2)$ (B) $(-1,2)$
(C) $(2,1)$ (D) $(0,0)$

116. The area enclosed by the curves $y = 4x^3$ and $y=16x$ is

(A) 32 (B) 16
(C) 64 (D) 2π

117. Two perpendicular tangents to $y^2 = 4ax$ always intersect on the line

(A) $x - a = 0$ (B) $x + a = 0$
(C) $x + 2a = 0$ (D) $x + 4a = 0$

118. The point which is equidistant from the points (0,0,0), (2,0,0), (0,2,0) and (2,2,2) is

(A) (1,0,1) (B) (0,1,0)
(C) (1.1.-1) (D) (1,1,1)

119. The volume of the parallelepiped whose edges are represented by the vectors $i + j,\ j + k,\ k + i$ is

(A) 2 (B) 0
(C) 1 (D) 6

120. Let $f(z) = \cos z - \frac{\sin z}{z}$ for non-zero $z \in \mathbb{C}$ and $f(0) = 0$, then $f(z)$ has a zero at

(A) $z = 0$ of order 1
(B) $z=0$ of order 2
(C) $z = 1$ of order 3
(D) $z=1$ of order 2

121. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, then $|\vec{a} + \vec{b} + \vec{c}|$ is

(A) $\sqrt{3}$
(B) 3
(C) 2
(D) 0

122. Let the characteristic equation of the matrix M be $\lambda^2 - \lambda - 1 = 0$, then

(A) M^{-1} does not exist
(B) M^{-1} exists but can't determine from the data
(C) $M^{-1} = M + 1$
(D) $M^{-1} = M - 1$

123. The maximum magnitude of the directional derivative for the surface $x^2 + xy + yz = 9$ at the point (1,2,3) is along the direction

(A) $\vec{i} + \vec{j} + \vec{k}$
(B) $\vec{i} + 2\vec{j} + 4\vec{k}$
(C) $\vec{i} + 2\vec{j} + \vec{k}$
(D) $\vec{i} + 2\vec{j} + 2\vec{k}$

124. The number of linearly independent eigen vectors of the matrix

$$\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$ is

(A) 1
(B) 2
(C) 3
(D) 4

125. The function $f(z) = \sqrt{|xy|}$ is

(A) everywhere analytic (B) nowhere analytic
(C) not analytic at origin (D) not continuous

126. x and y are two numbers and $x < y$. If $x^2 + y^2$ equals two times five and xy equals two times two, then $\frac{x}{y}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$
(C) 1 (D) 2

127. If $100 = x \pmod 7$, then least positive value of x is

(A) 7 (B) 4
(C) 3 (D) 2

128. The integral of $\int \frac{e^x}{1+e^{2x}} dx$ is

(A) $e^{2x} + C$ (B) $\tan^{-1}(e^x) + C$
(C) $\sec^{-1}(e^x) + C$ (D) $\log e^{2x} + C$

129. The value of $\int \tan x dx$ is

(A) $\log \sin x$ (B) $\log \cos x$
(C) $-\log \cos x$ (D) $\log \sec x$

130. $3 < x < 5$ is equivalent to

(A) $x = 4$ (B) $x = -4$
(C) $|x-4| < 1$ (D) $|x-4| > 1$

131. The equation $x^2 + y^2 - 2xy - 1 = 0$ represents

(A) a circle
(B) two perpendicular straight lines
(C) two parallel straight lines
(D) hyperbola

132. If $A = \{1,2,3,4,5\}$, then the number of proper subsets of A is

(A) 120
(B) 32
(C) 31
(D) 5

133. The value of $\int \frac{(x^2-1)}{(x^2+1)}\,dx$ is

(A) $\log(1+x^4)$
(B) $x - 2\tan^{-1}x$
(C) $x + 2\tan^{-1}x$
(D) $\tan^{-1}x^2$

134. The value of $\int_{-1}^{1} x|x|\,dx$ is

(A) 2
(B) 1
(C) ½
(D) 0

135. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx} =$

(A) $\sin\theta$
(B) $\frac{y}{x}$
(C) $\tan\theta$
(D) $\cos\theta$

136. If every element of a group G is its own inverse, then G *is*

(A) ring
(B) field
(C) cyclic group
(D) abelian group

137. For the differential equation of all straight lines passing through the origin $\frac{dy}{dx}$ is

(A) $\frac{y}{x}$
(B) $\frac{x}{y}$
(C) $x+y$
(D) $2(x-y)$

138. If $A=\{1,3,4,6\}$ and $B=\{1,9,16,36\}$ and a mapping f is defined by $f(a)=b;\ a \in A$ and $b \in B$ and $b=a^2$, then the mapping f is

(A) only injective mapping
(B) only surjective mapping
(C) bijective mapping
(D) None of these mapping

139. If $f:R \to R$ is defined by $f(x)=x^2$, then $f(x)$ is

(A) one-one
(B) onto
(C) both one-one and onto
(D) neither one-one nor onto

140. If $f(x)=\frac{(x-1)}{(x+1)}$, then $f[f\{f(x)\}]$ is equal to

(A) $-x$
(B) $\frac{-1}{x}$
(C) $-f(x)$
(D) $\frac{-1}{f(x)}$

141. The focus of the parabola $y^2-8x-32=0$ is at the point

(A) $(-2,0)$
(B) $(0,-2)$
(C) $(0,2)$
(D) $(4,0)$

142. A box contains 6 white and 4 black balls. If 5 balls are drawn at random from this box, the probability of getting exactly 3 white and 2 black balls is

(A) $\frac{1}{2}$ (B) $\frac{5}{6}$

(C) $\frac{12}{15}$ (D) None of the above

143. The imaginary part of $\sin(x+iy)$ is

(A) $\sin x \cosh y$ (B) $i \sin x \sinh y$

(C) $\cos x \cosh y$ (D) $\cos x \sinh y$

144. A number of 5 digits is formed with digits 1,2,3,4,5 without repetition. The probability that it is a number divisible by 4 is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

145. A student has to select 3 subjects out of 6 subjects - Mathematics, English, Hindi, Science, History and IT. If the student has chosen IT, the probability of choosing Mathematics is

(A) $\frac{1}{6}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{5}{6}$

146. Two values of x satisfying $-14 \equiv 6 \pmod{x}$ are

(A) 3, 4 (B) 4, 6

(C) 7, 10 (D) 2, 5

147. If a,b,c are in $G.P.$, then $\log_n a$, $\log_n b$ and $\log_n c$ are in

(A) A.P.
(B) G.P.
(C) H. P.
(D) None of the above

148. The angle between the straight lines $x^2 - y^2 - 2y - 1 = 0$ is

(A) 90°
(B) 75°
(C) 60°
(D) 35°

149. The integral of $\int_{-a}^{a} |x| dx$ is

(A) a^2
(B) $\frac{a^2}{4}$
(C) 0
(D) $-a^2$

150. $a * b = a \times b + b$ ($a, b \in N$ and $*$ is an operation on N).
Then $a * b = b * a$ implies

(A) $a = 0$
(B) $b = 0$
(C) $a = b$
(D) $a = -b$